Specification and Proof of Programs with Frama-C

SAC 2013 Tutorial

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Motivation

Main objective:

Rigorous, mathematical proof of semantic properties of a program

- functional properties
- safety:
    - all memory accesses are valid,
    - no arithmetic overflow,
    - no division by zero, . . .
- termination
- . . .

Our goal

In this tutorial, we will see

- how to specify a C program
- how to prove it with an automatic tool
- how to understand and fix proof failures

Outline

Outline

Introduction

- Frama-C tool
- ACSL specification language
- Jessie plugin

Function contracts

- Pre- and postconditions
- Specification with behaviors
- Contracts and function calls

Programs with loops

- Loop invariants
- Loop termination
- More exercises

My proof fails... What to do?

Conclusion

A brief history

- 90's: CAVEAT, a Hoare logic-based tool for C programs
- 2000's: CAVEAT used by Airbus during certification of the A380
- 2002: Why tool and its C front-end Caduceus
- 2006: Joint project to write a successor to CAVEAT and Caduceus
- 2008: First public release of Frama-C (Hydrogen)
- 2009: Hoare-logic based Frama-C plugin Jessie developed at INRIA
- 2012: New Hoare-logic based plugin WP developed at CEA LIST
- Frama-C today:
  - Most recent release: Frama-C Oxygen
  - Multiple projects around the platform
  - A growing community of users

Frama-C at a glance

- FRAmework for Modular Analysis of C programs
  - Various plugins: CFG, value analysis (abstract interpretation), impact analysis, dependency analysis, slicing, program proof, . . .
- Developed at CEA LIST and INRIA Saclay (Proval/Toccata team)
- Released under LGPL license
- Kernel based onCIL library [Necula et al. – Berkeley]
- Includes ACSL specification language
- Extensible platform
  - Adding specialized plugins is easy
  - Collaboration of analyses over the same code
  - Inter-plugin communication through ACSL formulas
- `http://frama-c.com/`

ACSL: ANSI/ISO C Specification Language

Presentation

- Based on the notion of contract, like in Eiffel
- Allows the users to specify functional properties of their programs
- Allows communication between various plugins
- Independent from a particular analysis
- ACSL manual at `http://frama-c.com/acsl`

Basic Components

- First-order logic
- Pure C expressions
- C types + $\mathbb{Z}$ (integer) and $\mathbb{R}$ (real)
- Built-ins predicates and logic functions, particularly over pointers: `\valid(p)` `\valid(p+0..2)`, `\separated(p+0..2,q+0..5)`, `\block_length(p)`

Jessie plugin

- Hoare-logic based plugin, developed at INRIA Saclay
- Proof of functional properties of the program
- Modular verification (function per function)
- Input: a program and a specification in ACSL
- Jessie generates verification conditions (VCs)
- Use of Automatic Theorem Provers to discharge the VCs
  - Alt-Ergo, Simplify, Z3, Yices, CVC3, . . .
- If all VCs are proved, the program respects the given specification
  - Does it mean that the program is correct?

Jessie plugin

- Hoare-logic based plugin, developed at INRIA Saclay
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- Input: a program and a specification in ACSL
- Jessie generates verification conditions (VCs)
- Use of Automatic Theorem Provers to discharge the VCs
  - Alt-Ergo, Simplify, Z3, Yices, CVC3, . . .
- If all VCs are proved, the program respects the given specification
  - Does it mean that the program is correct?
  - If the specification is wrong, the program can be wrong
- Limitations
  - Casts between pointers and integers
  - Limited support for union type
  - Aliasing requires some care

In this tutorial

In this tutorial we use

- Frama-C Carbon
- Jessie and Why 2.29
- Alt-Ergo 0.93

To run Jessie on a C program `file.c`

- `frama-c -jessie file.c`

All examples were also tested with

- Frama-C Nitrogen
- Jessie and Why 2.31
- Why3 0.73
- Alt-Ergo 0.95

Outline

- Introduction
  - Frama-C tool
  - ACSL specification language
  - Jessie plugin
- Function contracts
  - Pre- and postconditions
  - Specification with behaviors
  - Contracts and function calls
- Programs with loops
  - Loop invariants
  - Loop termination
  - More exercises
- My proof fails... What to do?
- Conclusion

Contracts

- Goal: specification of imperative functions
- Approach: give assertions (i.e. properties) about the functions
  - Precondition is supposed to be true on entry (ensured by callers of the function)
  - Postcondition must be true on exit (ensured by the function if it terminates)
- Nothing is guaranteed when the precondition is not satisfied
- Termination may or may not be guaranteed (total or partial correctness)

Primary role of contracts

- Main input of the verification process
- Must reflect the informal specification
- Should not be modified just to suit the verification tasks

Example 1

Specify and prove the following program:

```
// returns the absolute value of x
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

Example 1 (Continued) - Explain the proof failure

Explain the proof failure for the following program:

```
/*@ ensures (x >= 0 ==> \result == x) &&
          (x < 0 ==> \result == -x);
*/
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

Example 1 (Continued) - Explain the proof failure

Explain the proof failure for the following program:

```
/*@ ensures (x >= 0 ==> \result == x) &&
          (x < 0 ==> \result == -x);
*/
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

- For `x=INT_MIN`, `-x` cannot be represented by an `int` and overflows
- Example: on 32-bit, `INT_MIN`$= -2^{31}$ while `INT_MAX`$= 2^{31} - 1$

Safety warnings: arithmetic overflows

Absence of arithmetic overflows can be important to check

- A sad example: crash of Ariane 5 in 1996
- Jessie automatically generates VCs to check absence of overflows
- They ensure that arithmetic operations do not overflow
- If not proved, an overflow may occur. Is it intended?

Example 1 (Continued) - Solution

This is the completely specified program:

```
#include<limits.h>
/*@ requires x > INT_MIN;
    ensures (x >= 0 ==> \result == x) &&
      (x < 0 ==> \result == -x);
    assigns \nothing;
*/
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

Example 2

Specify and prove the following program:

```
// returns the maximum of x and y
int max ( int x, int y ) {
  if ( x >=y )
    return x ;
  return y ;
}
```

Example 2 (Continued) - Find the error

The following program is proved. Do you see any error?

```
/*@ ensures \result >= x && \result >= y;
*/
int max ( int x, int y ) {
  if ( x >=y )
    return x ;
  return y ;
}
```

Example 2 (Continued) - a wrong version

This is a wrong implementation that is also proved. Why?

```
#include<limits.h>
/*@ ensures \result >= x && \result >= y;
*/
int max ( int x, int y ) {
  return INT_MAX ;
}
```

Example 2 (Continued) - a wrong version

This is a wrong implementation that is also proved. Why?

```
#include<limits.h>
/*@ ensures \result >= x && \result >= y;
*/
int max ( int x, int y ) {
  return INT_MAX ;
}
```

- Our specification is incomplete
- Should say that the returned value is one of the arguments

Example 2 (Continued) - Solution

This is the completely specified program:

```
/*@ ensures \result >= x && \result >= y;
    ensures \result == x || \result == y;
    assigns \nothing;
*/
int max ( int x, int y ) {
  if ( x >=y )
    return x ;
  return y ;
}
```

Example 3

Specify and prove the following program:

```
// returns the maximum of *p and *q
int max_ptr ( int *p, int *q ) {
  if ( *p >= *q )
    return *p ;
  return *q ;
}
```

Example 3 (Continued) - Explain the proof failure

Explain the proof failure for the following program:

```
/*@ ensures \result >= *p && \result >= *q;
    ensures \result == *p || \result == *q;
*/
int max_ptr ( int *p, int *q ) {
  if ( *p >= *q )
    return *p ;
  return *q ;
}
```

Example 3 (Continued) - Explain the proof failure

Explain the proof failure for the following program:

```
/*@ ensures \result >= *p && \result >= *q;
    ensures \result == *p || \result == *q;
*/
int max_ptr ( int *p, int *q ) {
  if ( *p >= *q )
    return *p ;
  return *q ;
}
```

- Nothing ensures that pointers `p, q` are valid
- It must be ensured either by the function, or by its precondition

Safety warnings: invalid memory accesses

An invalid pointer or array access may result in a segmentation fault or memory corruption.

- Jessie automatically generates VCs to check memory access validity
- They ensure that each pointer (array) access has a valid offset (index)
- If the function assumes that an input pointer is valid, it must be stated in its precondition, e.g.
  - `\valid(p)` for one pointer `p`
  - `\valid(p+0..2)` for a range of offsets `p, p+1, p+2`

Example 3 (Continued) - Find the error

The following program is proved. Do you see any error?

```
/*@ requires \valid(p) && \valid(q);
    ensures \result >= *p && \result >= *q;
    ensures \result == *p || \result == *q;
*/
int max_ptr ( int *p, int *q ) {
  if ( *p >= *q )
    return *p ;
  return *q ;
}
```

Example 3 (Continued) - a wrong version

This is a wrong implementation that is also proved. Why?

```
/*@ requires \valid(p) && \valid(q);
    ensures \result >= *p && \result >= *q;
    ensures \result == *p || \result == *q;
*/
int max_ptr ( int *p, int *q ) {
  *p = 0;
  *q = 0;
  return 0 ;
}
```

Example 3 (Continued) - a wrong version

This is a wrong implementation that is also proved. Why?

```
/*@ requires \valid(p) && \valid(q);
    ensures \result >= *p && \result >= *q;
    ensures \result == *p || \result == *q;
*/
int max_ptr ( int *p, int *q ) {
  *p = 0;
  *q = 0;
  return 0 ;
}
```

- Our specification is incomplete
- Should say that the function cannot modify `*p` and `*q`

Frame rule

The clause `assigns v1, v2, ... , vN;`

- Part of the postcondition
- Specifies which (non local) variables can be modified by the function
- No need to specify local variable modifications in the postcondition
    - a function is allowed to change local variables
    - a postcondition cannot talk about them anyway, they do not exist after the function call
- Avoids to state that for any unchanged global variable `v`, we have `ensures \old(v) == v`
- Avoids to forget one of them: explicit permission is required
- If nothing can be modified, specify `assigns \nothing`

Example 3 (Continued) - Solution

This is the completely specified program:

```
/*@ requires \valid(p) && \valid(q);
    ensures \result >= *p && \result >= *q;
    ensures \result == *p || \result == *q;
    assigns \nothing;
*/
int max_ptr ( int *p, int *q ) {
  if ( *p >= *q )
    return *p ;
  return *q ;
}
```

Behaviors

Specification by cases

- Global precondition (`requires`) applies to all cases
- Global postcondition (`ensures`, `assigns`) applies to all cases
- Behaviors define contracts (refine global contract) in particular cases
- For each case (each `behavior`)
  - the subdomain is defined by `assumes` clause
  - the behavior's precondition is defined by `requires` clauses
    - it is supposed to be true whenever `assumes` condition is true
  - the behavior's postcondition is defined by `ensures`, `assigns` clauses
    - it must be ensured whenever `assumes` condition is true
- `complete behaviors` states that given behaviors cover all cases
- `disjoint behaviors` states that given behaviors do not overlap

Example 4

Specify using behaviors and prove the function `abs`:

```
// returns the absolute value of x
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

Example 4 (Continued) - Explain the proof failure for

```
#include<limits.h>
/*@ requires x > INT_MIN;
    assigns \nothing;
    behavior pos:
      assumes x > 0;
      ensures \result == x;
    behavior neg:
      assumes x < 0;
      ensures \result == -x;
    complete behaviors;
    disjoint behaviors;
*/
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

Example 4 (Continued) - Explain the proof failure for

```
#include<limits.h>
/*@ requires x > INT_MIN;
    assigns \nothing;
    behavior pos:
      assumes x > 0;
      ensures \result == x;
    behavior neg:
      assumes x < 0;
      ensures \result == -x;
    complete behaviors;
    disjoint behaviors;
*/
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

- The behaviors are not complete
- The case `x==0` is missing. A wrong value could be returned.

Example 4 (Continued) - Explain another proof failure for

```
#include<limits.h>
/*@ requires x > INT_MIN;
    assigns \nothing;
    behavior pos:
      assumes x >= 0;
      ensures \result == x;
    behavior neg:
      assumes x <= 0;
      ensures \result == -x;
    complete behaviors;
    disjoint behaviors;
*/
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

Example 4 (Continued) - Explain another proof failure for

```
#include<limits.h>
/*@ requires x > INT_MIN;
    assigns \nothing;
    behavior pos:
      assumes x >= 0;
      ensures \result == x;
    behavior neg:
      assumes x <= 0;
      ensures \result == -x;
    complete behaviors;
    disjoint behaviors;
*/
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

- The behaviors are not disjoint
- The case `x==0` is covered by both behaviors. Is it intended?

Example 4 (Continued) - Solution

```
#include<limits.h>
/*@ requires x > INT_MIN;
    assigns \nothing;
    behavior pos:
      assumes x >= 0;
      ensures \result == x;
    behavior neg:
      assumes x < 0;
      ensures \result == -x;
    complete behaviors;
    disjoint behaviors;
*/
int abs ( int x ) {
  if ( x >=0 )
    return x ;
  return -x ;
}
```

Contracts and function calls

Function calls are handled as follows:

- Suppose function g contains a call to a function `f`
- Suppose we try to prove the caller g
- Before the call to `f` in g, the precondition of `f` must be ensured by g
    - VCs is generated to prove that the precondition of `f` is respected
- After the call to `f` in g, the postcondition of `f` is supposed to be true
    - the postcondition of `f` is assumed in the proof below
    - modular verification: the code of `f` is not checked at this point
    - only a contract and a declaration of the callee `f` are required

Pre/post of the caller and of the callee have dual roles in the caller's proof

- Pre of the caller is supposed, Post of the caller must be ensured
- Pre of the callee must be ensured, Post of the callee is supposed

Example 5

Specify and prove the function `max_abs`

```
int abs ( int x );
int max ( int x, int y );

// returns maximum of absolute values of x and y
int max_abs( int x, int y ) {
  x=abs(x);
  y=abs(y);
  return max(x,y);
}
```

Example 5 (Continued) - Explain the proof failure for

```
#include<limits.h>
/*@ requires x > INT_MIN;
    ensures (x >= 0 ==> \result == x) &&
      (x < 0 ==> \result == -x);
    assigns \nothing; */
int abs ( int x );

/*@ ensures \result >= x && \result >= y;
    ensures \result == x || \result == y;
    assigns \nothing; */
int max ( int x, int y );

/*@ ensures \result >= x && \result >= -x &&
      \result >= y && \result >= -y;
    ensures \result == x || \result == -x ||
      \result == y || \result == -y;
    assigns \nothing; */
int max_abs( int x, int y ) {
  x=abs(x);
  y=abs(y);
  return max(x,y);
}
```

Example 5 (Continued) - Explain the proof failure for

```
#include<limits.h>
/*@ requires x > INT_MIN;
    ensures (x >= 0 ==> \result == x) &&
      (x < 0 ==> \result == -x);
    assigns \nothing; */
int abs ( int x );

/*@ ensures \result >= x && \result >= y;
    assigns \nothing; */
int max ( int x, int y );

/*@ requires x > INT_MIN;
    requires y > INT_MIN;
    ensures \result >= x && \result >= -x &&
      \result >= y && \result >= -y;
    ensures \result == x || \result == -x ||
      \result == y || \result == -y;
    assigns \nothing; */
int max_abs( int x, int y ) {
  x=abs(x);
  y=abs(y);
  return max(x,y);
}
```

Example 5 (Continued) - Solution

```
#include<limits.h>
/*@ requires x > INT_MIN;
    ensures (x >= 0 ==> \result == x) &&
      (x < 0 ==> \result == -x);
    assigns \nothing; */
int abs ( int x );

/*@ ensures \result >= x && \result >= y;
    ensures \result == x || \result == y;
    assigns \nothing; */
int max ( int x, int y );

/*@ requires x > INT_MIN;
    requires y > INT_MIN;
    ensures \result >= x && \result >= -x &&
      \result >= y && \result >= -y;
    ensures \result == x || \result == -x ||
      \result == y || \result == -y;
    assigns \nothing; */
int max_abs( int x, int y ) {
  x=abs(x);
  y=abs(y);
  return max(x,y);
}
```

Outline

Loops and automatic proof

- What is the issue with loops? Unknown, variable number of iterations
- The only possible way to handle loops: proof by induction
- Induction needs a suitable inductive property, that is proved to be
  - satisfied just before the loop, and
  - satisfied after $k+1$ iterations whenever it is satisfied after $k \geq 0$ iterations
- Such inductive property is called loop invariant
- The verification conditions for a loop invariant include two parts
  - loop invariant initially holds
  - loop invariant is preserved by any iteration

Loop invariants - some hints

How to find a suitable loop invariant? Consider two aspects:

- identify variables modified in the loop
  - variable number of iterations prevents from deducing their values (relationships with other variables)
  - define their possible value intervals (relationships) after *k* iterations
  - use `loop assigns` clause to list variables that (might) have been assigned so far after *k* iterations
- identify realized actions, or properties already ensured by the loop
  - what part of the job already realized after *k* iterations?
  - what part of the expected loop results already ensured after *k* iterations?
  - why the next iteration can proceed as it does? . . .

A stronger property on each iteration may be required to prove the final result of the loop

Some experience may be necessary to find appropriate loop invariants

Loop invariants - more hints

Remember: a loop invariant must be true

- before (the first iteration of) the loop, even if no iteration is possible
- after any complete iteration even if no more iterations are possible
- in other words, any time before the loop condition check

In particular, a `for` loop

```
for(i=0; i<n; i++) { /* body */ }
```

should be seen as

```
i=0;          // action before the first iteration
while( i<n ) // an iteration starts by the condition check
  {
    /* body */
    i++;      // last action in an iteration
  }
```

Example 6

Specify and prove the function `find_min`:

```
// returns the index of the minimal element
// of the given array a of size length
int find_min(int* a, int length) {
  int min, min_idx;
  min_idx = 0;
  min = a[0];
  for (int i = 1; i<length; i++) {
    if (a[i] < min) {
      min_idx = i;
      min = a[i];
    }
  }
  return min_idx;
}
```

Loop termination

- Program termination is undecidable
- A tool cannot deduce neither the exact number of iterations, nor even an upper bound
- If an upper bound is given, a tool can check it by induction
- An upper bound on the number of remaining loop iterations is the key idea behind the loop variant

Terminology

- Partial correctness: if the function terminates, it respects its specification
- Total correctness: the function terminates, and it respects its specification

Loop variants - some hints

- Unlike an invariant, a loop variant is an integer expression, not a predicate
- Loop variant is not unique: if V works, $V+1$ works as well
- No need to find a precise bound, any working loop variant is OK
- To find a variant, look at the loop condition
  - For the loop `while(exp1 > exp2 )`, try `loop variant exp1-exp2;`
- In more complex cases: ask yourself why the loop terminates, and try to give an integer upper bound on the number of remaining loop iterations

Example 6 (Continued) - Solution

```
/*@ requires length > 0 && \valid(a+(0..length-1));
    assigns \nothing;
    ensures 0<=\result<length &&
      (\forall integer j; 0<=j<length ==> a[\result]<=a[j]);*/
int find_min(int* a, int length) {
  int min, min_idx;
  min_idx = 0;
  min = a[0];
  /*@ loop invariant 0<=i<=length && 0<=min_idx<length;
      loop invariant \forall integer j; 0<=j<i ==> min<=a[j];
      loop invariant a[min_idx]==min;
      loop assigns min, min_idx, i;
      loop variant length - i; */
  for (int i = 1; i<length; i++) {
    if (a[i] < min) {
      min_idx = i;
      min = a[i];
    }
  }
  return min_idx;
}
```

Example 7

Specify and prove the function `all_zeros`:

```
// returns a non-zero value iff all elements
// in a given array t of n integers are zeros
int all_zeros(int t[], int n) {
  int k;
  for(k = 0; k < n; k++)
    if (t[k] != 0)
      return 0;
  return 1;
}
```

Example 7 (Continued) - Find the errors

```
/*@ requires n>=0 && \valid(t+(0..n-1));
    ensures \result != 0 <==>
      (\forall integer j; 0 <= j < n ==> t[j] == 0);
*/
int all_zeros(int t[], int n) {
  int k;
  /*@ loop invariant 0 <= k < n;
      loop variant n-k;
  */
  for(k = 0; k < n; k++)
    if (t[k] != 0)
      return 0;
  return 1;
}
```

Example 7 (Continued) - Solution

```
/*@ requires n>=0 && \valid(t+(0..n-1));
    assigns \nothing;
    ensures \result != 0 <==>
      (\forall integer j; 0 <= j < n ==> t[j] == 0);
*/
int all_zeros(int t[], int n) {
  int k;
  /*@ loop invariant 0 <= k <= n;
      loop invariant \forall integer j; 0<=j<k ==> t[j]==0;
      loop variant n-k;
  */
  for(k = 0; k < n; k++)
    if (t[k] != 0)
      return 0;
  return 1;
}
```

\forall and \exists - hints and examples

- Do not confuse `&&` and `==>` inside `\forall` and `\exists`
- Some common patterns:
  - `\forall integer j; 0 <= j && j < n ==> t[j] == 0;`
  - `\exists integer j; 0 <= j && j < n && t[j] != 0;`
  - Each one here is negation of the other
- A shorter form:
  - `\forall integer j; 0 <= j < n ==> t[j] == 0;`
  - `\exists integer j; 0 <= j < n && t[j] != 0;`
- With several variables:
  - `\forall integer i,j; 0 <= i <= j < length ==> a[i]<=a[j];`
  - `\exists integer i,j; 0 <= i <= j < length && a[i]>a[j]`

Example 8

Specify and prove the function `binary_search`:

```
/* takes as input a sorted array a, its length,
   and a value key to search,
   returns the index of a cell which contains key,
   returns -1 iff key is not present in the array
*/
int binary_search(int* a, int length, int key) {
  int low = 0, high = length - 1;
  while (low<=high) {
    int mid = (low+high)/2;
    if (a[mid] == key) return mid;
    if (a[mid] < key) { low = mid+1; }
    else { high = mid - 1; }
  }
  return -1;
}
```

Example 8 (Continued) - Solution (1/2)

```
/*@ predicate sorted{L}(int* a, int length) =
      \forall integer i,j; 0<=i<=j<length ==> a[i]<=a[j];
*/
/*@ requires \valid(a+(0..length -1));
    requires sorted(a,length);
    requires length >=0;

    assigns \nothing;

    behavior exists:
      assumes \exists integer i; 0<=i<length && a[i] == key;
      ensures 0<=\result<length && a[\result] == key;

    behavior not_exists:
      assumes \forall integer i; 0<=i<length ==> a[i] != key;
      ensures \result == -1;

    complete behaviors;
    disjoint behaviors;
*/
```

Example 8 (Continued) - Solution (2/2)

```
int binary_search(int* a, int length, int key) {
  int low = 0, high = length - 1;
  /*@ loop invariant 0<=low<=high+1;
      loop invariant high<length;
      loop assigns low,high;
      loop invariant \forall integer k; 0<=k<low ==> a[k] < key;
      loop invariant \forall integer k; high<k<length ==> a[k] > key;
      loop variant high-low;
  */
  while (low<=high) {
    int mid = low+(high-low)/2;
    if (a[mid] == key) return mid;
    if (a[mid] < key) { low = mid+1; }
    else { high = mid - 1; }
  }
  return -1;
}
```

Example 9

Specify and prove the function `sort`:

```
// sorts given array a of size length > 0
void sort (int* a, int length) {
  int current;
  for (current = 0; current < length - 1; current++) {
    int min_idx = current;
    int min = a[current];
    for (int i = current + 1; i < length; i++) {
      if (a[i] < min) {
        min = a[i];
        min_idx = i;
      }
    }
    if (min_idx != current){
      L: a[min_idx]=a[current];
      a[current]=min;
    }
  }
}
```

Referring to another state

- Specification may require values at differents program points
- Use `\at(e,L)` to refer to the value of expression `e` at label `L`
- Some predefined labels:
  - `\at(e,Here)` refers to the current state
  - `\at(e,Old)` refers to the pre-state
  - `\at(e,Post)` refers to the post-state
- `\old(e)` is equivalent to `\at(e,Old)`

Example 9 (Continued) - Solution (1/3)

```
/*@ predicate sorted{L}(int* a, integer length) =
      \forall integer i,j; 0<=i<=j<length ==> a[i]<=a[j];
*/
/*@ predicate swap{L1,L2}(int* a,integer i,integer j,integer length)=
      0<=i<j<length
      && \at(a[i],L1) == \at(a[j],L2)
      && \at(a[i],L2) == \at(a[j],L1)
      && \forall integer k; 0<=k<length && k!=i && k!=j ==>
           \at(a[k],L1) == \at(a[k],L2);
*/
/*@ inductive same_elements{L1,L2}(int*a , integer length) {
      case refl{L}:
        \forall int*a, integer length; same_elements{L,L}(a,length);
      case swap{L1,L2}: \forall int*a, integer i,j,length;
        swap{L1,L2}(a,i,j,length) ==> same_elements{L1,L2}(a,length);
      case trans{L1,L2,L3}: \forall int*a, integer length;
        same_elements{L1,L2}(a,length)
        ==> same_elements{L2,L3}(a,length)
        ==> same_elements{L1,L3}(a,length);
}
*/
```

Example 9 (Continued) - Solution (2/3)

```
/*@ requires \valid(a+(0..length-1));
    requires length > 0;
    assigns a[0..length-1];
    behavior sorted:
      ensures sorted(a,length);
    behavior same_elements:
      ensures same_elements{Pre,Here}(a,length);
*/
void sort (int* a, int length) {
  int current;
  /*@ loop invariant 0<=current<length;
      loop assigns a[0..length-1],current;
      for sorted: loop invariant sorted(a,current);
      for sorted: loop invariant
        \forall integer i,j; 0<=i<current<=j<length ==> a[i] <= a[j];
      for same_elements: loop invariant
        same_elements{Pre,Here}(a,length);
      loop variant length-current;
   */
```

Example 9 (Continued) - Solution (3/3)

```
  for (current = 0; current < length - 1; current++) {
    int min_idx = current;
    int min = a[current];
    /*@ loop invariant current+1<=i<=length;
        loop assigns i,min,min_idx;
        loop invariant current<=min_idx<i;
        loop invariant a[min_idx] == min;
        for sorted: loop invariant
          \forall integer j; current<=j<i ==> min <= a[j];
        loop variant length -i;
    */
    for (int i = current + 1; i < length; i++) {
      if (a[i] < min) {
        min = a[i];
        min_idx = i;
      }
    }
    if(min_idx != current) {
      L: a[min_idx]=a[current];
      a[current]=min;
 /*@for same_elements:assert swap{L,Here}(a,current,min_idx,length);*/
    }
  }
}
```

Outline

Proof failures

A proof of a VC for some annotation can fail for various reasons:

- incorrect implementation (→ check your code)
- incorrect annotation (→ check your spec)
- missing or erroneous (previous) annotation (→ check your spec)
- insufficient timeout (→ try longer timeout)
- complex property that automatic provers cannot handle.

Analysis of proof failures

When a proof failure is due to the specification, the erroneous annotation may be not obvious to find. For example:

- proof of a "loop invariant preserved" may fail in case of
    - incorrect loop invariant
    - incorrect loop invariant in a previous, or inner, or outer loop
    - missing `assumes` or `loop assumes` clause
    - too weak precondition
    - ...
- proof of a postcondition may fail in case of
    - incorrect loop invariant (too weak, too strong, or inappropriate)
    - missing `assumes` or `loop assumes` clause
    - inappropriate postcondition in a called function
    - too weak precondition
    - ...

Analysis of proof failures (Continued)

- Additional statements (`assert`, `lemma`, . . .) may help the prover
  - They can be provable by the same (or another) prover or checked elsewhere
- Separating independent properties (e.g. in separate, non disjoint behaviors) may help
  - The prover may get lost with a bigger set of hypotheses (some of which are irrelevant)

When nothing else helps to finish the proof:

- an interactive proof assistant can be used
- Coq, Isabelle, PVS, are not that scary: we may need only a small portion of the underlying theory

Outline

Introduction
- Frama-C tool
- ACSL specification language
- Jessie plugin

Function contracts
- Pre- and postconditions
- Specification with behaviors
- Contracts and function calls

Programs with loops
- Loop invariants
- Loop termination
- More exercises

My proof fails... What to do?

Conclusion

Conclusion

- We learned how to specify and prove a C program with Frama-C
- Hoare-logic based tools provide a powerful way to formally verify programs
- The program is proved with respect to the given specification, so
  - Absence of proof failures is not sufficient
  - The specification must be correct
- The proof is automatic, but analysis of proof failures is manual
- Proof failures help to complete the specification or find bugs
- Interactive proof tools may be necessary to finish the proof for complex properties that cannot be proved automatically