Basic ACSL Features

Jochen Burghardt

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Overview

From unit tests to a function contract

Function to be tested:

```
// returns the index of an occurrence of v in a[]

extern int find(int n,const int a[n],int v);
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Unit test 1:

```
const int const a1[5] = { 9,7,8,9,6 };
int const f1 = find(5,a1,8);
assert(f1 == 2);
```

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Unit test 2:

```
const int const a1[5] = { 9,7,8,9,6 };
int const f2 = find(5,a1,15);
assert(f2 ==   );
```

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Unit test 2:

```
const int const a1[5] = { 9,7,8,9,6 };
int const f2 = find(5,a1,15);
assert(f2 == -1);
```

(Informal description needs to be refined)

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Unit test 3:

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const int const a1[5] = { 9,7,8,9,6 };
int const f3 = find(5,a1,9);
assert(f3 == 0);
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(Search order used by `find()` is unknown)

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Unit test assertions:

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Create a unique `assert` expression working for all tests
(It will be used as ASCL function contract in verification)

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May cause exception when `find() < 0` or `find() >= n`

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Does not consider “not-found” case

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Wrong implementation always returning -1 would pass

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We therefor now switch to ACSL

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- Since our code is no longer valid C code; we will now fully convert it to proper ACSL

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- We need to restrict the range of `a[]` in the first `ensures`

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- Frama-C also ignores both

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- Most C compilers ignore an upper-bound expression in an array declaration
- Many C compilers ignore the `const` keyword
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Completing a function contract for `find()`: entry conditions

```
// returns the index of an occurrence of v in a[], or -1
/*@ requires 0 <= n && \valid_range(a,0,n-1);
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- This is stronger than the `const`, but still true and useful in our example

Summary: writing a function contract in ACSL

- A function contract formally describes what a function is expected to do
- It may be considered as a unique `assert` expression usable for any unit test of the function
- To describe function contracts, ACSL provides a more powerful language than C

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Function contracts: Frama-C vs. <assert.h>

Generic function to be tested / verified:

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extern T0 func(T1 x1,...Tk xk);
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Corresponding test-code:

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setup(&x1,...,&xk);     // obtain sample input data
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    T0 const x0 = func(x1,...,xk);
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Note:

- `requires true` means: `func` runs on all inputs

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Note:

- Some ACSL expressions cannot be translated to C , e.g. `\forall`

Programming vs. specification language

Program	**Specification**
Commands to be executed `i = 0;`	Descriptions of a particular state `/*@ i == 0 */`
~~`i == 0;`~~ Warning: Statement without effect	~~`/*@ i = 0 */`~~ Error: Side effect in specification

`double sqrt(double x) {`
`  // Newton's algorithm`
`  // (too complicated`
`  //  to be shown here)`
`}`
Commands to obtain the square-root
	`/*@ requires x >= 0.0;`
	`  ensures  1.0e-12 >=`
	`  fabs(\result*\result-x);`
	`*/`
	
	Test to cross-check the obtained result

Function contract language summary: Junctors

ACSL expression		meaning	
`!p`	negation	as in C:	`(p?false:true)`
`p&&q`	conjunction	as in C:	`(p?q:false)`
`p||q`	disjunction	as in C:	`(p?true:q)`
`p==>q`	implication	`(p?q:true)`	or similarly: `(!p)||q`
`p<==>q`	equivalence	`p==q`	

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`p<==>q`	equivalence	`p==q`	

Examples:

```
/*@ensures !(\exists int i; 0<=i<n&&a[i]==v) ==> \result==-1;
   ensures  (\exists int i; 0<=i<n&&a[i]==v) ==> a[\result]==v;
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extern int find(int n,const int a[n],int v);
```

Function contract language summary: Quantifiers

Universal quantifier:

```
\forall int x; p(x)
```

is equivalent to an infinite conjunction:

```
...&& p(-2) && p(-1) && p(0) && p(1) && p(2) && p(3) && ...
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- Order of quantifiers **does** matter:
  `\forall man_t m;  \exists woman_t w; loves(m,w)` vs.
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  `\forall man_t m;  \exists woman_t w; loves(m,w)` vs.
  `\exists woman_t w; \forall man_t m;  loves(m,w)`
- Unique existence **cannot** be expressed with `\exists`:
  "p holds at least once": `\exists int i;  p(i)`
  "p holds at most once": `!(\exists int i,j; p(i) && p(j) && i!=j)`
  or equivalently: `\forall int i,j; p(i) && p(j) ==> i==j`

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  ```
  \forall int i; 0<=i<8 ==> a[i] != NULL
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- Use ==> to restrict the range of a universal quantifier:
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- Use && to restrict the range of an existential quantifier:
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Verifying a function implementation against a contract

```
/*@ requires 0 <= n && \valid_range(a,0,n-1);
    assigns \nothing;
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- 5 conditions couldn't be verified in our first attempt

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- We run Frama-C / Jessie on find.c to generate a verification-condition file
- We run the Simplify prover on the verification-condition file
- 5 conditions couldn't be verified in our first attempt
- We try to eliminate the reasons, starting with the easiest one ("variant decreases")

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- Frama-C tries to prove the termination of each loop

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- More formally, an expression `E` such that always `E>0` and `E` properly decreases with every loop cycle
- We choose `n-i`

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- For simple loops like in our example, the choice of `E` might be automated in future
- In general, automatically chosing an appropriate `E` is impossible for theoretical reasons [Tur36]

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```

- Running Frama-C / Jessie / Simplify again, we note that 4 unproven conditions remain
- We now consider "pointer dereferencing"
- Frama-C couldn't verify that `i>=0` when computing `a[i]`

Verifying a function implementation against a contract

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int find(int n,const int a[n],int v)
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    /*@ loop   variant n - i;
    */
    for (i=0; i<n; ++i)
        if (a[i] == v)
            return i;
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- Frama-C couldn't verify that `i>=0` when computing `a[i]`
- In fact, a tiny induction proof is needed for this:

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- Frama-C couldn't verify that `i>=0` when computing `a[i]`
- In fact, a tiny induction proof is needed for this:
  - `i>=0` holds after 0 loop cycles, i.e. immediately after the assignment `i=0`;
  - if `i>=0` holds after k loop cycles, then it also holds after $k+1$ cycles, since `i` is increased only

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- Since theorem provers are still unable to guess inductive invariants, we have to tell Frama-C explicitely that `i>=0`
- We use a `loop invariant` clause for this end
- We write "`i<=n`" to catch also the very last cycle

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- We run Frama-C / Jessie / Simplify again, and only one unproven condition remains, referring to the first postcondition (`ensures`)

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- We have to tell Frama-C the core idea of our algorithm:

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- We have to tell Frama-C the core idea of our algorithm:
  "no element of `a[]` inspected so far matched `v`"

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}
```

- We have to tell Frama-C the core idea of our algorithm:
  "no element of `a[]` inspected so far matched `v`"
  formally: `\forall int j; 0<=j<i ==> a[j] != v;`

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- We use another `loop invariant` clause for this

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- We run Frama-C / Jessie / Simplify again, and now everything could be proved

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- Our implementation of `find` has been verified

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- Our implementation of `find` has been verified
- Every unit test of `find` on arguments satisfying the `requires` clause is guaranteed to pass an `assert` macro corresponding to the `ensures` clause

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- Why not "`0<=j<=i`" in the loop invariant?

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- Why not "`0<=j<=i`" in the loop invariant?
- This depends on where the loop invariants holds: before or after the incrementing `++i`

Where does a loop invariant hold?

```
// statements before loop
//@ loop invariant
for (  i=0;   i<n;   ++i  ) {
   // body statements
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// statements after loop
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- Control flow of a `for` loop

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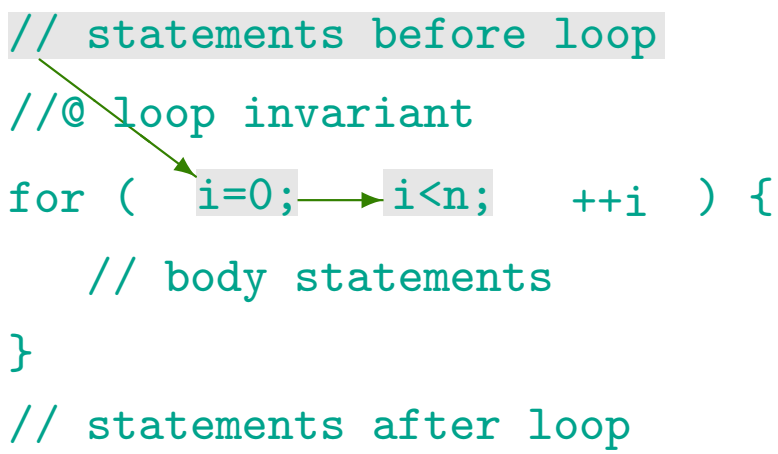

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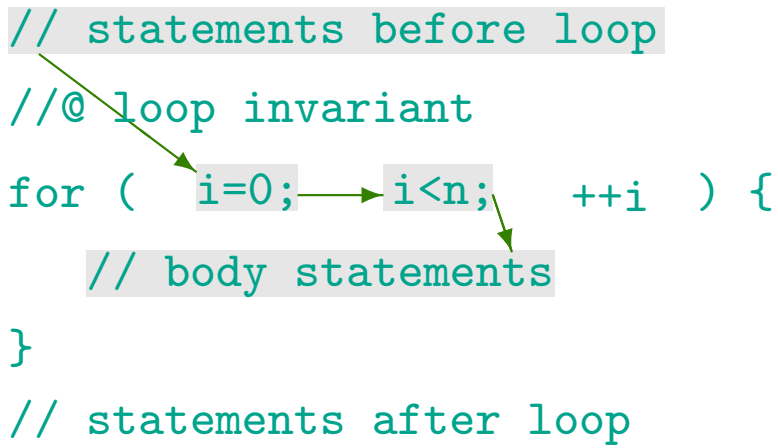

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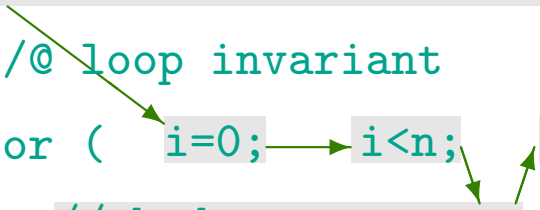

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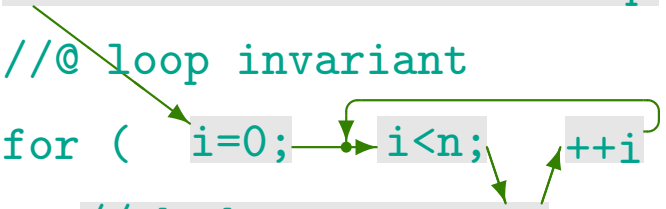

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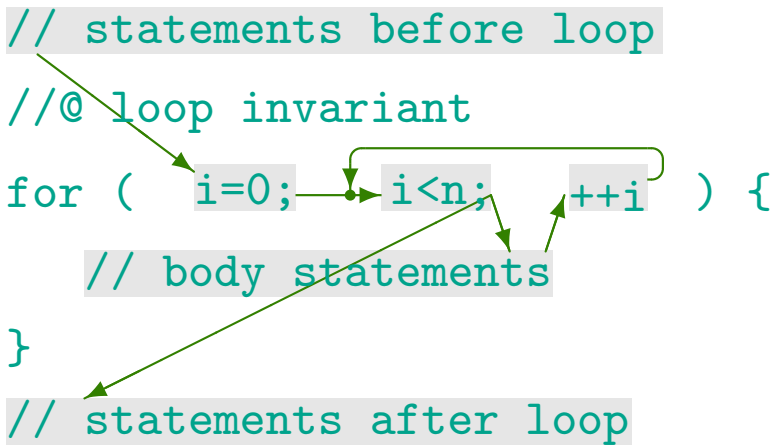
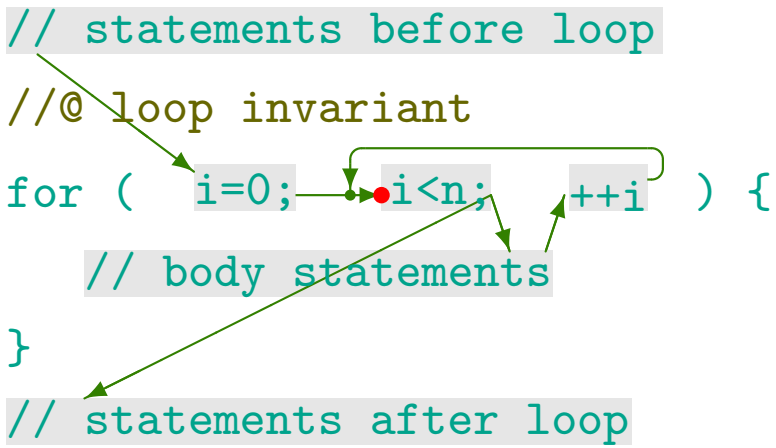


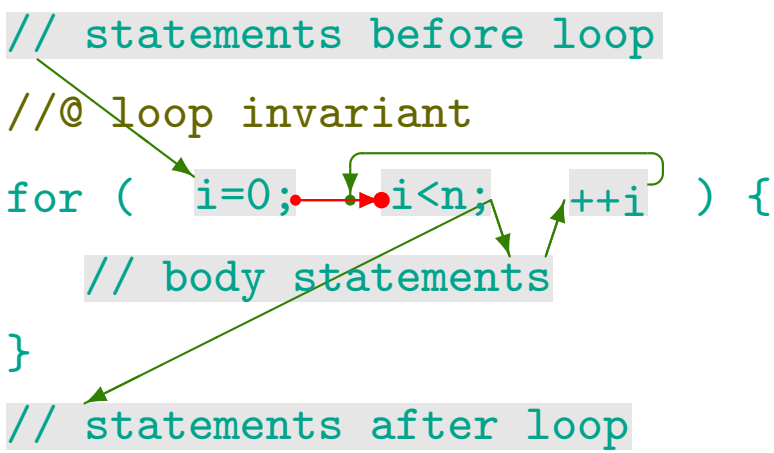
- Control flow of a `for` loop
- "*Where*" means: "*at which arcs of the control flow graph*" rather than: "*at which line/column of the source text*"

Where does a loop invariant hold?



- The loop invariant has to hold immediately before `test`,

Where does a loop invariant hold?



- The loop invariant has to hold immediately before `test`,
- hence also immediately after `init`

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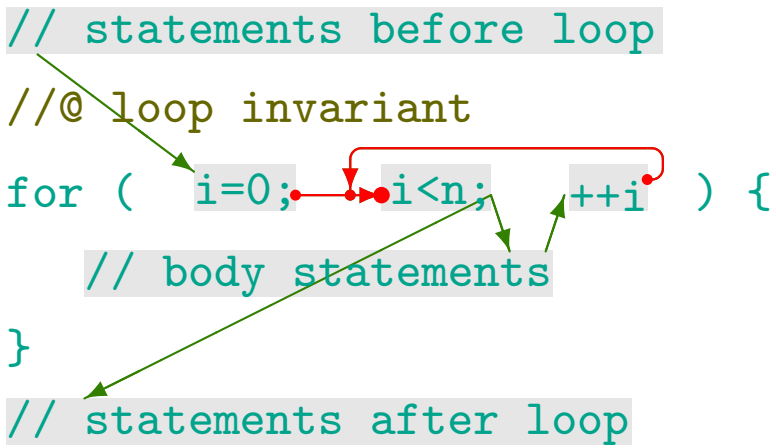
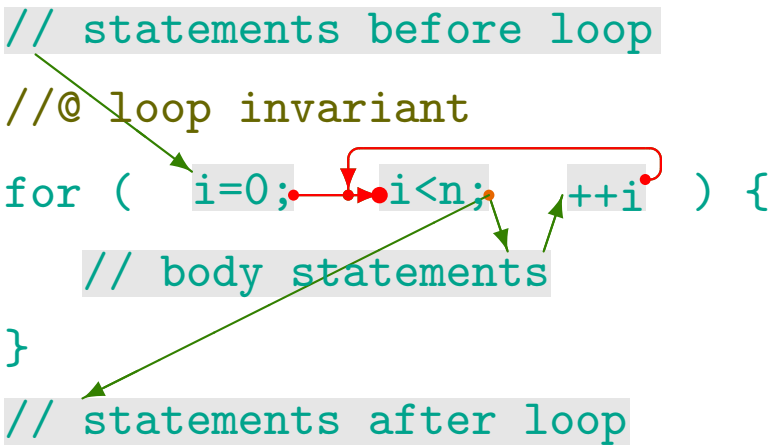


- The loop invariant has to hold immediately before `test`,
- hence also immediately after `init` and `incr`.

Where does a loop invariant hold?



- The loop invariant has to hold immediately before `test`,
- hence also immediately after `init` and `incr`.
- If `test` has no side effects,
  the loop invariant has to hold also immediately after `test`,

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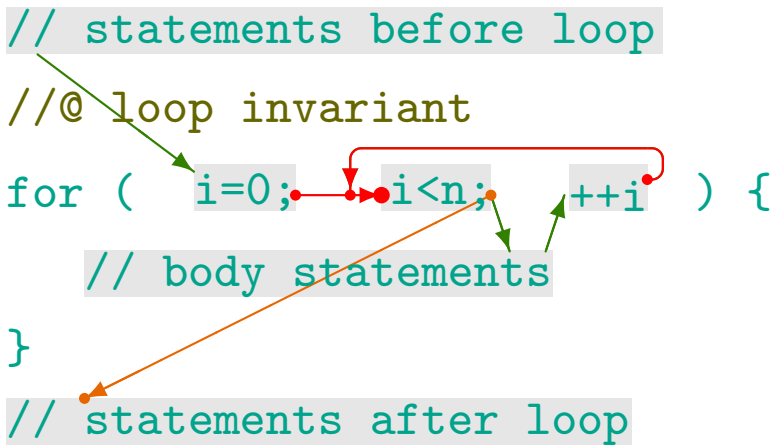

- The loop invariant has to hold immediately before `test`,
- hence also immediately after `init` and `incr`.
- If `test` has no side effects,
  the loop invariant has to hold also immediately after `test`,
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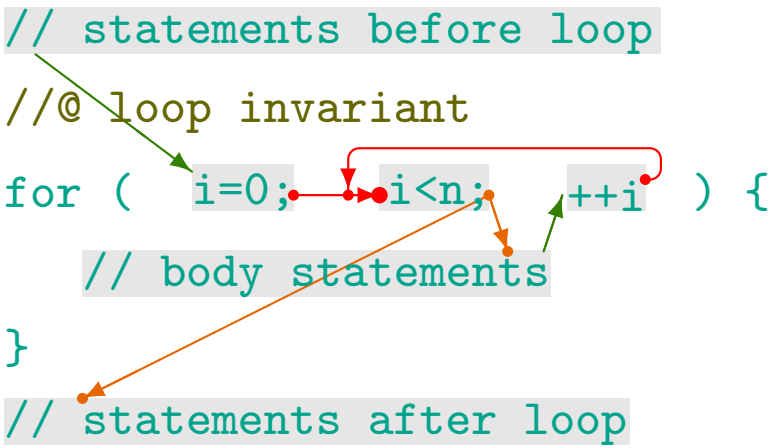

- The loop invariant has to hold immediately before `test`,
- hence also immediately after `init` and `incr`.
- If `test` has no side effects,
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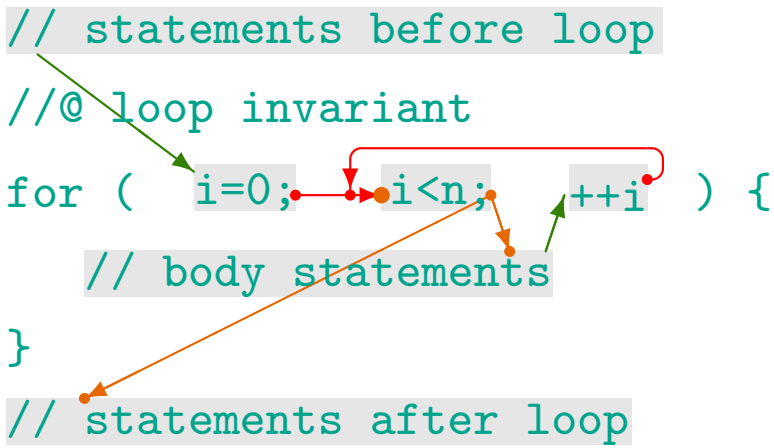
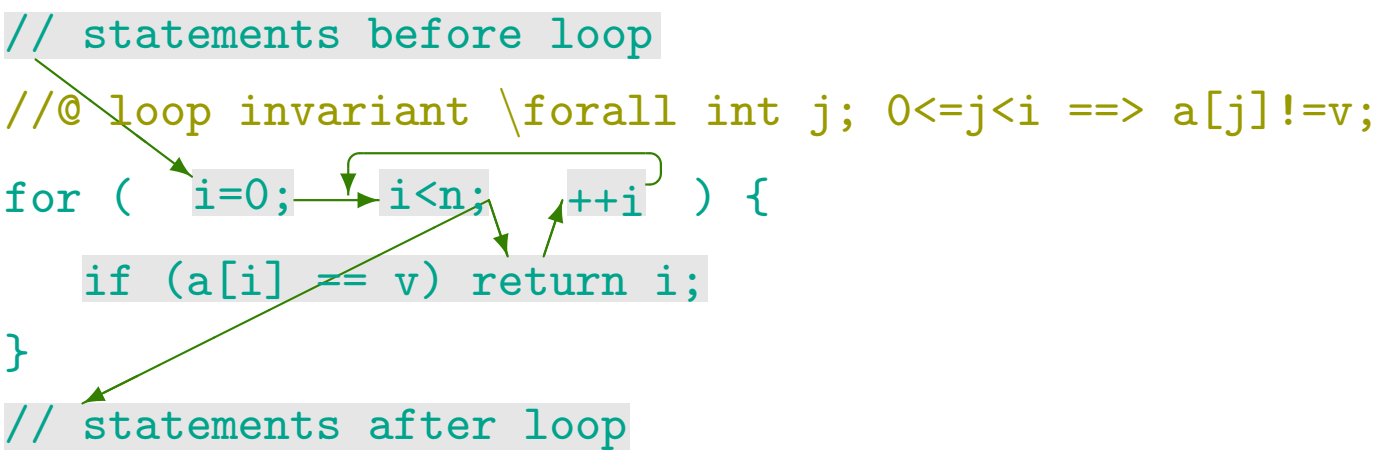


- The loop invariant has to hold immediately before `test`,
- hence also immediately after `init` and `incr`.
- If `test` has no side effects,
  the loop invariant has to hold also immediately after `test`,
- hence also immediately after the loop and before `body`.
- It does usually **not** hold before `init`, inside or after `body` or before `incr`!

Where does a loop invariant hold?



- Considering our `find` example, its 2nd loop invariant

Where does a loop invariant hold?

```
// statements before loop
//@ loop invariant \forall int j; 0<=j<i ==> a[j]!=v;
for (   i=0;      i<n;      ++i   ) {
    if (a[i] == v) return i;
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// statements after loop
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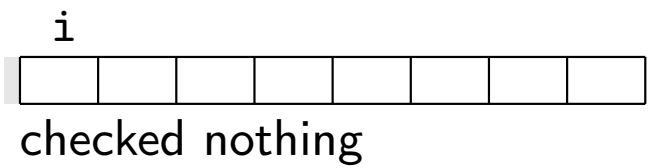
- Considering our `find` example, its 2nd loop invariant
- adequately describes the state after `init`



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- adequately describes the state after `init`
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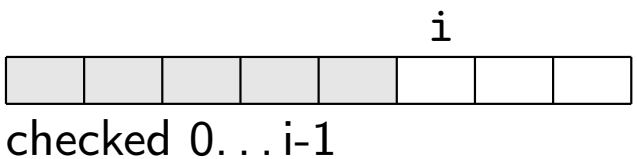

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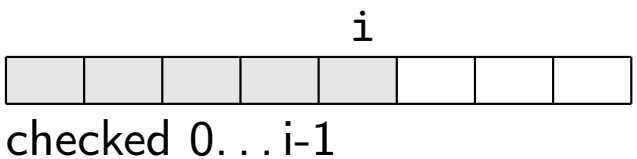
- Considering our `find` example, its 2nd loop invariant
- adequately describes the state after `init`
- and after `incr`,
- but it would be too weak to describe the state before `incr`.



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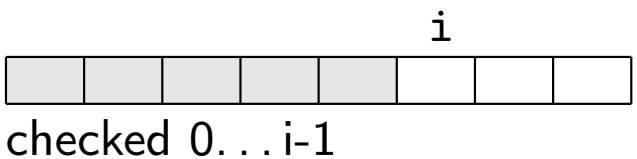

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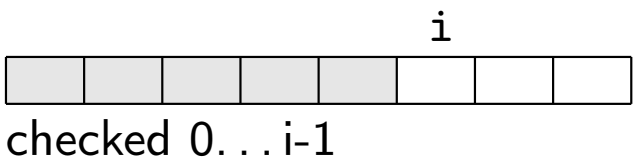
- Considering our `find` example, its 2nd loop invariant
- adequately describes the state after `init`
- and after `incr`.
- If the loop terminates normally,



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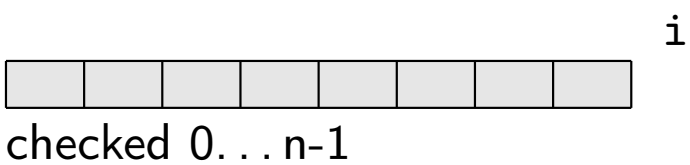
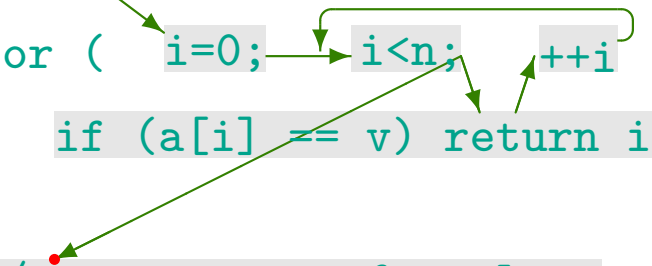
- Considering our `find` example, its 2nd loop invariant
- adequately describes the state after `init`
- and after `incr`.
- If the loop terminates normally,
- we know that all array elements have been checked (without success)



Where does a loop invariant hold?

```
// statements before loop
//@ loop invariant \forall int j; 0<=j<i ==> a[j]!=v;
for (   i=0;      i<n;      ++i   ) {
    if (a[i] == v) return i;
}
// statements after loop
```



- Considering our `find` example, its 2nd loop invariant
- adequately describes the state after `init`
- and after `incr`.
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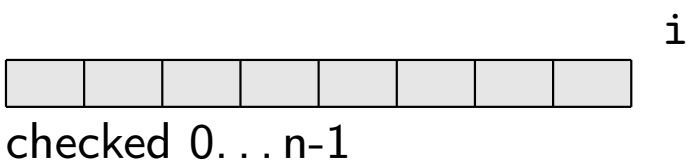

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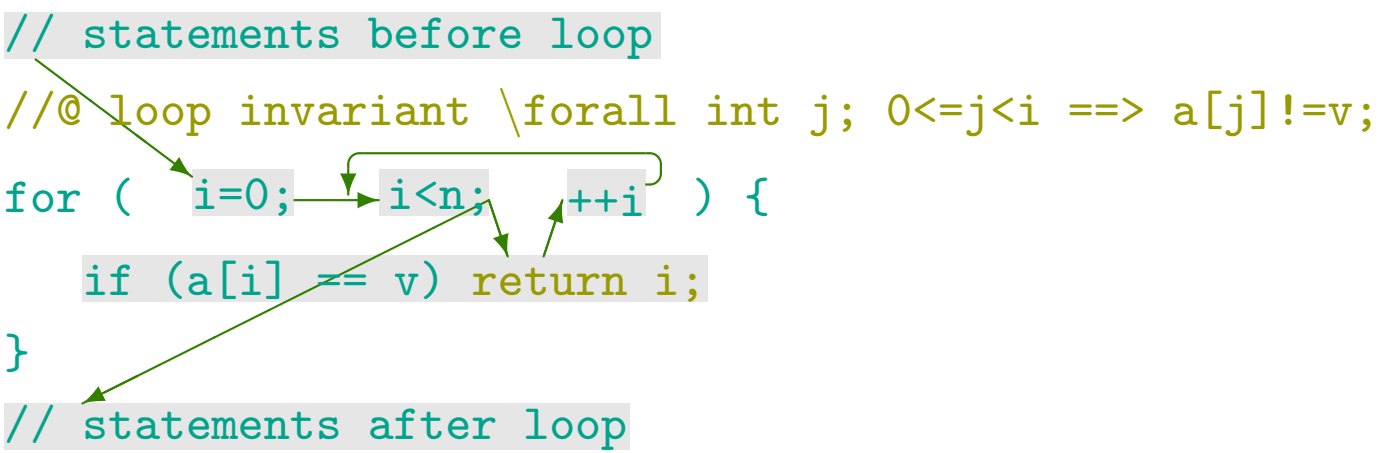

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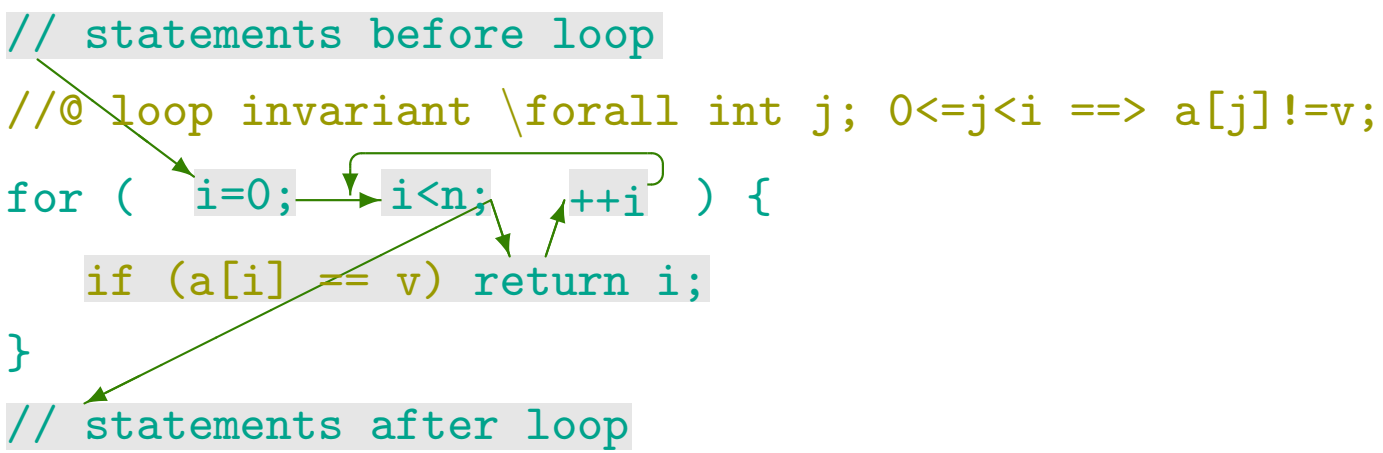

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- If the loop terminates normally,
- we know that all array elements have been checked (without success)
- and hence returning -1 will satisfy our contract.
- If the loop is terminated by the `return`,
- we obtain the postcondition directly from the preceding `if`.

References in `requires` and `ensures` clauses

```
double updateMean(int *sum,int *cnt,int new)
{
    *sum += new;
    *cnt += 1;
    return (double)*sum / (double)*cnt;      // mean value so far
}
```

- Example: incrementally computing the mean value of an `int` sequence

References in `requires` and `ensures` clauses

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Another example: Euklid's gcd algorithm

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/*@ requires A > 0 && B > 0;
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        div(i,A) && div(i,B) <==> div(i,\result);
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int gcd(int A, int B) {
    int const a = A, b = B;
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- Shows complementary set of `C`:
  `while` loop, numeric computation,
  nontrivial `loop variant`, infinite quantifier range

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- Example computation:

a:	**72**	**30**		**18**	**6**
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- Contract: "The divisors of `\result` are exactly the common divisors of `A` and `B`"
- `\result` is the greatest divisor of itself, and hence the greatest common divisor of `A` and `B`

Predicate definition of `div`

```
/*@ axiomatic Euklid {
    predicate div(integer a, integer b) =
        (0 < a && 0 <= b && b % a == 0);
    axiom A1:
        \forall integer a, b, c;
            b >= c && div(a,b) && div(a,c) ==> div(a,b-c);
    axiom A2:
        \forall integer a, b, c;
            b >= c && div(a,c) && div(a,b-c) ==> div(a,b);
    }
*/
```

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