Development of security-critical software with SPARK/Ada at secunet

Stefan Berghofer

30.5.2017

Agenda

1. Introduction
2. A Link between Why3 and Isabelle
3. Applications
4. Conclusion

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1. **Introduction**
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About secunet Security Networks AG

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- One of Germany's leading providers of IT security

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- More details: `www.secunet.com`

Context

High-security VPN gateways and clients

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- Process various categories of data with different classification

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- Prove at least absence of runtime exceptions for all trusted components, for some also functional correctness

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SPARK at secunet

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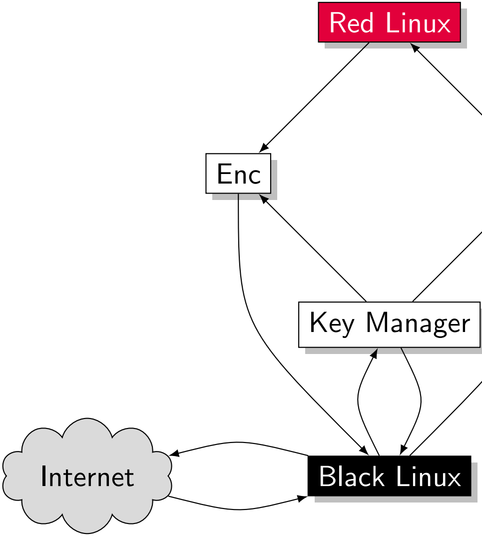
2017 Proof of properties of Muen separation kernel (ongoing)

Component-Based Architecture



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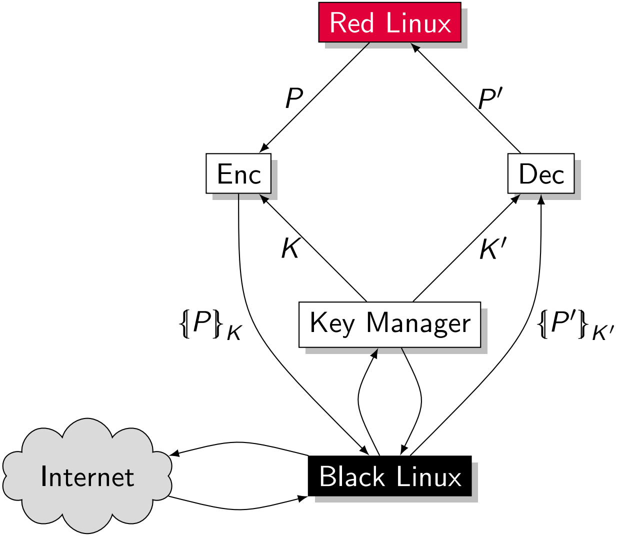

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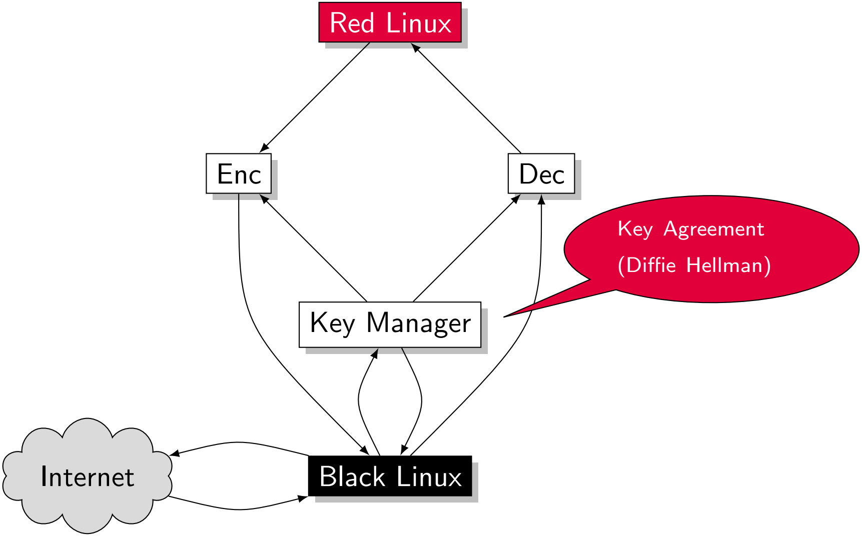

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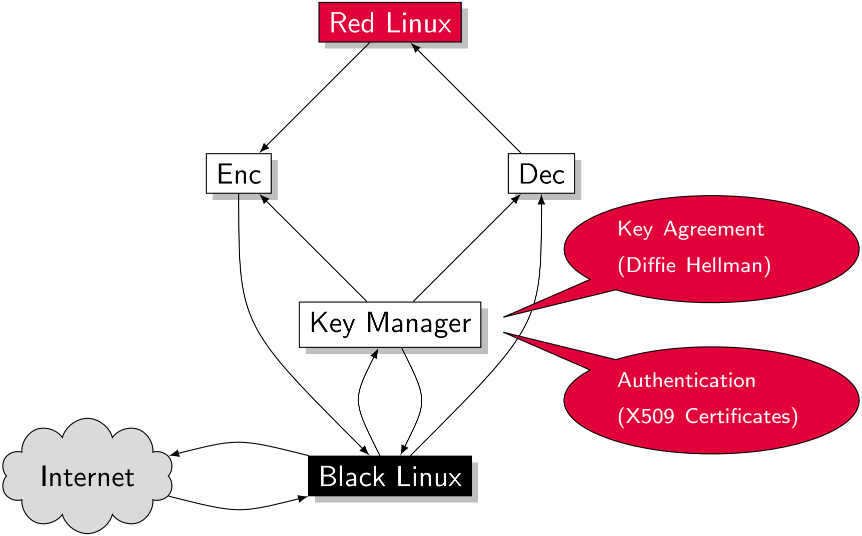

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  use lemma subprograms to help automatic provers

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- Complex external specifications can be linked to SPARK code using ghost functions

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    New concepts must be introduced using already existing and more primitive concepts.

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- More information: `isabelle.in.tum.de`

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- System keeps track of proved / unproved VCs

SPARK Toolchain

Source Files
(*.ads, *.adb)

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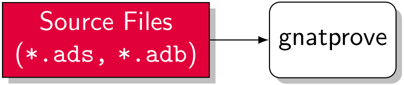
SPARK Toolchain



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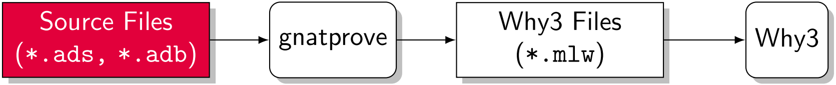

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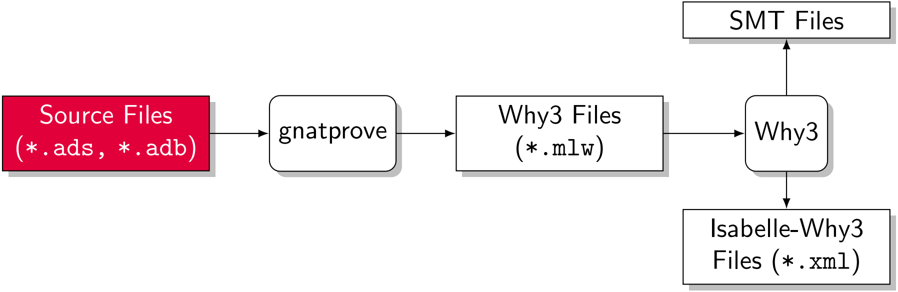

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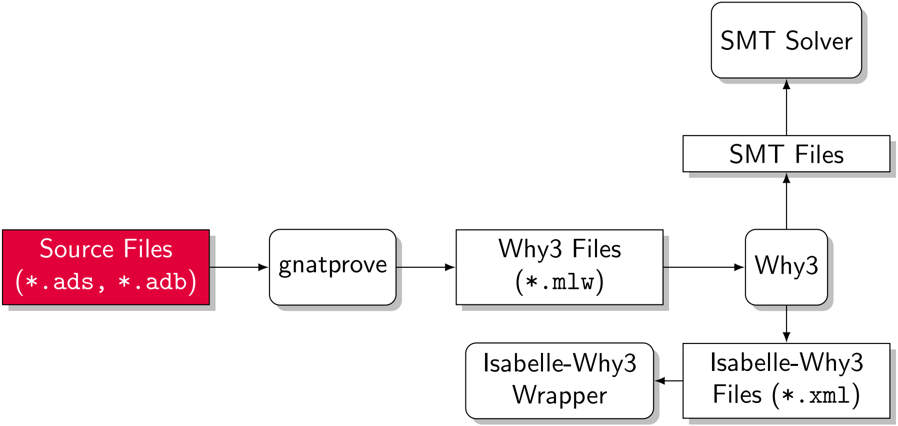

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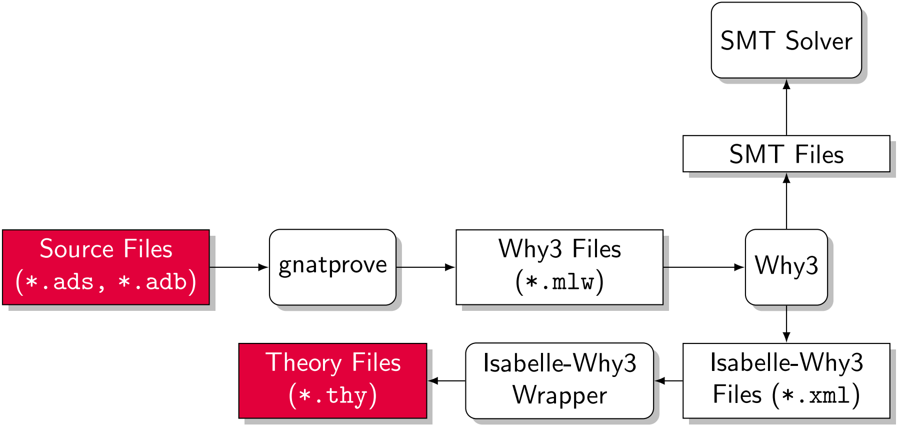

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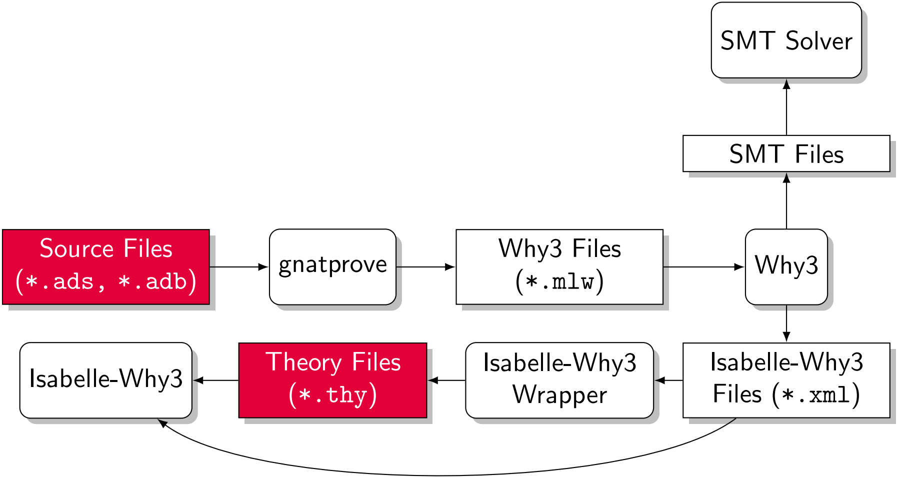

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Example: Euclidean Algorithm

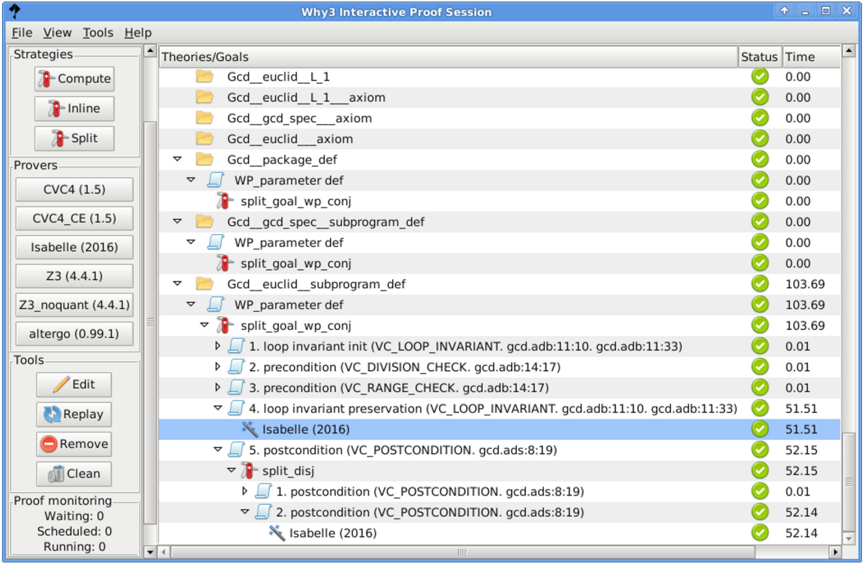
```
function GCD_Spec (M, N : Natural) return Natural
  with Ghost, Import;

function Euclid (M, N : Natural) return Natural
  with Post => Euclid'Result = GCD_Spec (M, N)
is
   A, B, R : Natural;
begin
   A := M; B := N;

   loop
      pragma Loop_Invariant (GCD_Spec (A, B) = GCD_Spec (M, N));
      exit when B = 0;
      R := A mod B;
      A := B; B := R;
   end loop;

   return A;
end Euclid;
```

Managing VCs using Why3 IDE



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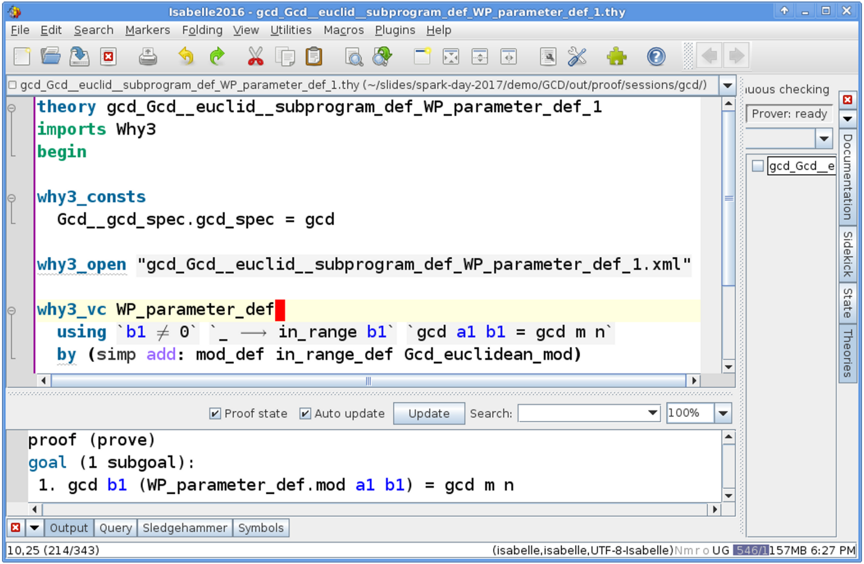
Proving VCs using Isabelle



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Commands of Why3-Plugin for Isabelle

why3_open Load and parse VCs

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why3_consts etc. can be viewed as light-weight on-the-fly variant of Why3's realizations that happen completely on the Isabelle side

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libsparkcrypto – A Cryptographic Library for SPARK

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Open Source

`http://git.codelabs.ch/?p=spark-crypto.git`

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Layers of Cryptographic Functions

Big numbers	Modular multiplication, addition, subtraction

Layers of Cryptographic Functions

Elliptic curves (basic)	Point addition and doubling
Big numbers	Modular multiplication, addition, subtraction

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Elliptic curves (derived)	Scalar multiplication
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Layers of Cryptographic Functions

Security protocols	ECDSA, ECDH
Elliptic curves (derived)	Scalar multiplication
Elliptic curves (basic)	Point addition and doubling
Big numbers	Modular multiplication, addition, subtraction

Basic Declarations for Big Numbers

```
package Bignum
is

   function Base return Math_Int.Math_Int is (Math_Int.From_Word32 (2) ** 32)
     with Ghost;

   subtype Big_Int_Range is Natural range Natural'First .. Natural'Last - 1;

   type Big_Int is array (Big_Int_Range range <>) of Types.Word32;

   function Num_Of_Big_Int (A : Big_Int; F, L : Natural)
     return Math_Int.Math_Int
     with Ghost, Import, Global => null;

   function Num_Of_Boolean (B : Boolean) return Math_Int.Math_Int
     with Ghost, Import, Global => null;

   function Inverse (M, A : Math_Int.Math_Int) return Math_Int.Math_Int
     with Ghost, Import, Global => null;
                                              ...
end Bignum;
```

Formalization of Big Numbers in Isabelle

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Formalization of Big Numbers in Isabelle

Abstraction function

$$\textit{num-of-big-int} :: (\textit{int} \Rightarrow \textit{int}) \Rightarrow \textit{int} \Rightarrow \textit{int} \Rightarrow \textit{int}$$
$$\textit{num-of-big-int}\ A\ k\ i = (\sum j = 0..{<}i.\ \textit{Base}^{j} * A\ (k + j))$$

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Summation property

$$\mathit{num\text{-}of\text{-}big\text{-}int}\ A\ k\ (i + j) =$$
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Modular inverse

$$\mathit{minv} :: \mathit{int} \Rightarrow \mathit{int} \Rightarrow \mathit{int}$$
$$\mathit{coprime}\ x\ m \Longrightarrow 0 < x \Longrightarrow 1 < m \Longrightarrow x * \mathit{minv}\ m\ x\ \mathit{mod}\ m = 1$$

Montgomery Multiplication

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Computes

$$x \otimes y = x \cdot y \cdot R^{-1} \bmod m \qquad \text{where } R = b^n$$

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Perform computations on numbers in Montgomery format

$$\widetilde{x} = x \cdot R \bmod m \qquad \text{(likewise for } \widetilde{y}\text{)}$$

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Multiplication of numbers in Montgomery format

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$$\widetilde{x} \otimes \widetilde{y} = x \cdot R \cdot y \cdot R \cdot R^{-1} \bmod m = x \cdot y \cdot R \bmod m = \widetilde{x \cdot y}$$

Conversion between standard and Montgomery format

$$x \otimes (R^2 \bmod m) = x \cdot R^2 \cdot R^{-1} \bmod m = x \cdot R \bmod m = \widetilde{x}$$

$$\widetilde{x} \otimes 1 = x \cdot R \cdot 1 \cdot R^{-1} \bmod m = x \bmod m$$

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n - 1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

Efficiently Computing $456 \cdot 789 \bmod 987$

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end if

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	$0 \quad +$

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end for
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  $a \leftarrow a - m$
end if

	0	+
$6 \cdot 789$	4734	=

$-7^{-1} \bmod 10 = 7$

secunet

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$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
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end if

$-7^{-1} \bmod 10 = 7$

	0	$+$
$6 \cdot 789$	4734	$=$
	4734	$+$

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end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	+
$6 \cdot 789$	4734	=
	4734	+
$8 \cdot 987$	7896	=

secunet

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	+
$6 \cdot 789$	4734	=
	4734	+
$8 \cdot 987$	7896	=
	12630	/ 10 =

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	+
$6 \cdot 789$	4734	=
	4734	+
$8 \cdot 987$	7896	=
	12630	/ 10 =
	1263	+

secunet

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	$+$
$6 \cdot 789$	4734	$=$
	4734	$+$
$8 \cdot 987$	7896	$=$
	12630	$/\ 10\ =$
	1263	$+$
$5 \cdot 789$	3945	$=$

Efficiently Computing $456 \cdot 789 \bmod 987$

$$
\begin{array}{l}
a \leftarrow 0 \\
\textbf{for } i = 0 \textbf{ to } n-1 \textbf{ do} \\
\quad u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b \\
\quad a \leftarrow (a + x_i \cdot y + u \cdot m)/b \\
\textbf{end for} \\
\textbf{if } a \geq m \textbf{ then} \\
\quad a \leftarrow a - m \\
\textbf{end if}
\end{array}
$$

$$-7^{-1} \bmod 10 = 7$$

	0	+
$6 \cdot 789$	4734	=
	4734	+
$8 \cdot 987$	7896	=
	12630	/ 10 =
	1263	+
$5 \cdot 789$	3945	=
	5208	+

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	$+$
$6 \cdot 789$	4734	$=$
	4734	$+$
$8 \cdot 987$	7896	$=$
	12630	$/\ 10\ =$
	1263	$+$
$5 \cdot 789$	3945	$=$
	5208	$+$
$6 \cdot 987$	5922	$=$

secunet

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	$+$
$6 \cdot 789$	4734	$=$
	4734	$+$
$8 \cdot 987$	7896	$=$
	12630	$/\ 10\ =$
	1263	$+$
$5 \cdot 789$	3945	$=$
	5208	$+$
$6 \cdot 987$	5922	$=$
	11130	$/\ 10\ =$

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	$+$
$6 \cdot 789$	4734	$=$
	4734	$+$
$8 \cdot 987$	7896	$=$
	12630	$/\ 10 =$
	1263	$+$
$5 \cdot 789$	3945	$=$
	5208	$+$
$6 \cdot 987$	5922	$=$
	11130	$/\ 10 =$
	1113	$+$

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	+
$6 \cdot 789$	4734	=
	4734	+
$8 \cdot 987$	7896	=
	12630	/ 10 =
	1263	+
$5 \cdot 789$	3945	=
	5208	+
$6 \cdot 987$	5922	=
	11130	/ 10 =
	1113	+
$4 \cdot 789$	3156	=

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	+	
$6 \cdot 789$	4734	=	
	4734	+	
$8 \cdot 987$	7896	=	
	12630	/ 10	=
	1263	+	
$5 \cdot 789$	3945	=	
	5208	+	
$6 \cdot 987$	5922	=	
	11130	/ 10	=
	1113	+	
$4 \cdot 789$	3156	=	
	4269	+	

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Efficiently Computing $456 \cdot 789 \bmod 987$

$$
\begin{aligned}
& a \leftarrow 0 \\
& \textbf{for } i = 0 \textbf{ to } n-1 \textbf{ do} \\
& \quad u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b \\
& \quad a \leftarrow (a + x_i \cdot y + u \cdot m)/b \\
& \textbf{end for} \\
& \textbf{if } a \geq m \textbf{ then} \\
& \quad a \leftarrow a - m \\
& \textbf{end if}
\end{aligned}
$$

$-7^{-1} \bmod 10 = 7$

$$
\begin{array}{r|rl}
 & 0 & + \\
6 \cdot 789 & 4734 & = \\
 & 4734 & + \\
8 \cdot 987 & 7896 & = \\
 & 12630 & /\ 10\ = \\
\hline
 & 1263 & + \\
5 \cdot 789 & 3945 & = \\
 & 5208 & + \\
6 \cdot 987 & 5922 & = \\
 & 11130 & /\ 10\ = \\
\hline
 & 1113 & + \\
4 \cdot 789 & 3156 & = \\
 & 4269 & + \\
3 \cdot 987 & 2961 & = \\
 & & \\
\hline
 & &
\end{array}
$$

Efficiently Computing $456 \cdot 789 \bmod 987$

$$
\begin{array}{l}
a \leftarrow 0 \\
\textbf{for } i = 0 \textbf{ to } n-1 \textbf{ do} \\
\quad u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b \\
\quad a \leftarrow (a + x_i \cdot y + u \cdot m)/b \\
\textbf{end for} \\
\textbf{if } a \geq m \textbf{ then} \\
\quad a \leftarrow a - m \\
\textbf{end if}
\end{array}
$$

$-7^{-1} \bmod 10 = 7$

$$
\begin{array}{r|rl}
 & 0 & + \\
6 \cdot 789 & 4734 & = \\
 & 4734 & + \\
8 \cdot 987 & 7896 & = \\
 & 12630 & /\ 10\ = \\
\hline
 & 1263 & + \\
5 \cdot 789 & 3945 & = \\
 & 5208 & + \\
6 \cdot 987 & 5922 & = \\
 & 11130 & /\ 10\ = \\
\hline
 & 1113 & + \\
4 \cdot 789 & 3156 & = \\
 & 4269 & + \\
3 \cdot 987 & 2961 & = \\
 & 7230 & /\ 10\ = \\
\hline
 & &
\end{array}
$$

secunet

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n - 1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

	0	+
$6 \cdot 789$	4734	=
	4734	+
$8 \cdot 987$	7896	=
	12630	/ 10 =
	1263	+
$5 \cdot 789$	3945	=
	5208	+
$6 \cdot 987$	5922	=
	11130	/ 10 =
	1113	+
$4 \cdot 789$	3156	=
	4269	+
$3 \cdot 987$	2961	=
	7230	/ 10 =
	723	

Efficiently Computing $456 \cdot 789 \bmod 987$

$a \leftarrow 0$
for $i = 0$ **to** $n-1$ **do**
  $u \leftarrow (a_0 + x_i \cdot y_0) \cdot -m_0^{-1} \bmod b$
  $a \leftarrow (a + x_i \cdot y + u \cdot m)/b$
end for
if $a \geq m$ **then**
  $a \leftarrow a - m$
end if

$-7^{-1} \bmod 10 = 7$

$723 \cdot 1000 \bmod 987 = 516 =$
$456 \cdot 789 \bmod 987$

	0	+	
$6 \cdot 789$	4734	=	
	4734	+	
$8 \cdot 987$	7896	=	
	12630	/ 10	=
	1263	+	
$5 \cdot 789$	3945	=	
	5208	+	
$6 \cdot 987$	5922	=	
	11130	/ 10	=
	1113	+	
$4 \cdot 789$	3156	=	
	4269	+	
$3 \cdot 987$	2961	=	
	7230	/ 10	=
	723		

Montgomery Multiplication in SPARK

```
for I in Natural range A_First .. A_Last
loop
   Carry1 := 0; Carry2 := 0;
   XI := X (X_First + (I - A_First));
   U := (A (A_First) + XI * Y (Y_First)) * M_Inv;
   Single_Add_Mult_Mult
     (A (A_First), XI, Y (Y_First),
      M (M_First), U, Carry1, Carry2);
   Add_Mult_Mult
     (A, A_First, A_Last - 1,
      Y, Y_First + 1, M, M_First + 1,
      XI, U, Carry1, Carry2);
   A (A_Last) := A_MSW + Carry1;
   A_MSW := Carry2 + Word_Of_Boolean (A (A_Last) < Carry1);
end loop;
```

Specification of Montgomery Multiplication

Preconditions

```
Num_Of_Big_Int (Y, Y_First, A_Last - A_First + 1) <
Num_Of_Big_Int (M, M_First, A_Last - A_First + 1) and
1 < Num_Of_Big_Int (M, M_First, A_Last - A_First + 1) and
1 + M_Inv * M (M_First) = 0
```

Specification of Montgomery Multiplication

Preconditions

```
Num_Of_Big_Int (Y, Y_First, A_Last - A_First + 1) <
Num_Of_Big_Int (M, M_First, A_Last - A_First + 1) and
1 < Num_Of_Big_Int (M, M_First, A_Last - A_First + 1) and
1 + M_Inv * M (M_First) = 0
```

Postcondition

```
Num_Of_Big_Int (A, A_First, A_Last - A_First + 1) =
(Num_Of_Big_Int (X, X_First, A_Last - A_First + 1) *
 Num_Of_Big_Int (Y, Y_First, A_Last - A_First + 1) *
 Inverse (Num_Of_Big_Int (M, M_First, A_Last - A_First + 1),
   Base) ** (A_Last - A_First + 1)) mod
Num_Of_Big_Int (M, M_First, A_Last - A_First + 1)
```

Specification of Montgomery Multiplication

Preconditions

```
Num_Of_Big_Int (Y, Y_First, A_Last - A_First + 1) <
Num_Of_Big_Int (M, M_First, A_Last - A_First + 1) and
1 < Num_Of_Big_Int (M, M_First, A_Last - A_First + 1) and
1 + M_Inv * M (M_First) = 0
```

Postcondition

```
Num_Of_Big_Int (A, A_First, A_Last - A_First + 1) =
(Num_Of_Big_Int (X, X_First, A_Last - A_First + 1) *
 Num_Of_Big_Int (Y, Y_First, A_Last - A_First + 1) *
 Inverse (Num_Of_Big_Int (M, M_First, A_Last - A_First + 1),
   Base) ** (A_Last - A_First + 1)) mod
Num_Of_Big_Int (M, M_First, A_Last - A_First + 1)
```

In mathematical notation...

$$a = x \cdot y \cdot b^{-n} \bmod m$$

Elliptic Curves

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- Applications: ECDH (key agreement) and ECDSA (authentication)

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  - Projective (x, y, z)

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- Abstract properties formalized in Isabelle based on Coq formalization by Laurent Théry
- Proved correspondence of SPARK implementation with abstract specification

Elliptic Curves – Abstract Specification

$$\textbf{datatype}\ 'a\ point = Infinity \mid Point\ 'a\ 'a$$

$$\textbf{locale}\ \mathit{ell\text{-}field} = \mathit{field} +$$
$$\quad \textbf{assumes}\ \mathit{two\text{-}not\text{-}zero}\text{: } \ll 2 \gg \neq \mathbf{0}$$

$$\textbf{definition}\ \mathit{nonsingular} :: 'a \Rightarrow 'a \Rightarrow \mathit{bool}\ \textbf{where}$$
$$\quad \mathit{nonsingular}\ a\ b = (\ll 4 \gg \otimes a \uparrow 3 \oplus \ll 27 \gg \otimes b \uparrow 2 \neq \mathbf{0})$$

$$\textbf{definition}\ \mathit{on\text{-}curve} :: 'a \Rightarrow 'a \Rightarrow 'a\ point \Rightarrow \mathit{bool}\ \textbf{where}$$
$$\quad \mathit{on\text{-}curve}\ a\ b\ p = (\mathit{case}\ p\ \mathit{of}$$
$$\qquad Infinity \Rightarrow True$$
$$\qquad \mid Point\ x\ y \Rightarrow x \in \mathit{carrier}\ R \wedge y \in \mathit{carrier}\ R \wedge$$
$$\qquad\quad y \uparrow 2 = x \uparrow 3 \oplus a \otimes x \oplus b)$$

Point Addition

$$
\begin{array}{l}
\textbf{definition}\ add :: {}'a \Rightarrow {}'a\ point \Rightarrow {}'a\ point \Rightarrow {}'a\ point\ \textbf{where} \\
\quad add\ a\ p_1\ p_2 = (case\ p_1\ of \\
\qquad\quad Infinity \Rightarrow p_2 \\
\qquad \mid Point\ x_1\ y_1 \Rightarrow (case\ p_2\ of \\
\qquad\qquad Infinity \Rightarrow p_1 \\
\qquad\quad \mid Point\ x_2\ y_2 \Rightarrow \\
\qquad\qquad\quad if\ x_1 = x_2\ then \\
\qquad\qquad\qquad if\ y_1 = \ominus\ y_2\ then\ Infinity \\
\qquad\qquad\qquad else\ let\ l = (\ll 3 \gg \otimes x_1 \uparrow 2 \oplus a) \oslash (\ll 2 \gg \otimes y_1); \\
\qquad\qquad\qquad\qquad\quad x_3 = l \uparrow 2 \ominus \ll 2 \gg \otimes x_1 \\
\qquad\qquad\qquad\qquad in\ Point\ x_3\ (\ominus\ y_1 \ominus l \otimes (x_3 \ominus x_1)) \\
\qquad\qquad\quad else\ let\ l = (y_2 \ominus y_1) \oslash (x_2 \ominus x_1); \\
\qquad\qquad\qquad\qquad x_3 = l \uparrow 2 \ominus x_1 \ominus x_2 \\
\qquad\qquad\qquad in\ Point\ x_3\ (\ominus\ y_1 \ominus l \otimes (x_3 \ominus x_1))))
\end{array}
$$

Point Addition – Properties

lemma *add-closed*:
assumes $a \in carrier\ R$ **and** $b \in carrier\ R$
and $on\text{-}curve\ a\ b\ p_1$ **and** $on\text{-}curve\ a\ b\ p_2$
shows $on\text{-}curve\ a\ b\ (add\ a\ p_1\ p_2)$

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  assumes $a \in carrier\ R$ **and** $b \in carrier\ R$
  and $on\text{-}curve\ a\ b\ p_1$ **and** $on\text{-}curve\ a\ b\ p_2$
  shows $on\text{-}curve\ a\ b\ (add\ a\ p_1\ p_2)$

lemma *add-comm*:
  assumes $a \in carrier\ R$ **and** $b \in carrier\ R$
  and $on\text{-}curve\ a\ b\ p_1$ **and** $on\text{-}curve\ a\ b\ p_2$
  shows $add\ a\ p_1\ p_2 = add\ a\ p_2\ p_1$

Point Addition – Properties

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assumes $a \in carrier\ R$ **and** $b \in carrier\ R$
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shows $on\text{-}curve\ a\ b\ (add\ a\ p_1\ p_2)$

lemma *add-comm*:
assumes $a \in carrier\ R$ **and** $b \in carrier\ R$
and $on\text{-}curve\ a\ b\ p_1$ **and** $on\text{-}curve\ a\ b\ p_2$
shows $add\ a\ p_1\ p_2 = add\ a\ p_2\ p_1$

lemma *add-assoc*:
assumes a: $a \in carrier\ R$ **and** b: $b \in carrier\ R$
and ab: $nonsingular\ a\ b$
and p_1: $on\text{-}curve\ a\ b\ p_1$ **and** p_2: $on\text{-}curve\ a\ b\ p_2$
and p_3: $on\text{-}curve\ a\ b\ p_3$
shows $add\ a\ p_1\ (add\ a\ p_2\ p_3) = add\ a\ (add\ a\ p_1\ p_2)\ p_3$

Projective Coordinates

type-synonym $'a\ ppoint = {'a} \times {'a} \times {'a}$

definition (**in** *field*) *make-affine* :: $'a\ ppoint \Rightarrow {'a}\ point$ **where**
make-affine $p =$
$(let\ (x, y, z) = p$
$in\ if\ z = \mathbf{0}\ then\ Infinity\ else\ Point\ (x \oslash z)\ (y \oslash z))$

lemma (**in** *ell-field*) *padd-correct*:
assumes a: $a \in carrier\ R$ **and** b: $b \in carrier\ R$
and p_1: $on\text{-}curvep\ a\ b\ p_1$ **and** p_2: $on\text{-}curvep\ a\ b\ p_2$
shows $make\text{-}affine\ (padd\ a\ p_1\ p_2) =$
$add\ a\ (make\text{-}affine\ p_1)\ (make\text{-}affine\ p_2)$

Projective Coordinates – Equality

definition (**in** *cring*) *proj-eq* :: $'a$ *ppoint* $\Rightarrow$ $'a$ *ppoint* $\Rightarrow$ *bool*
where

$$
\begin{aligned}
&\textit{proj-eq} = (\lambda(x_1, y_1, z_1)\ (x_2, y_2, z_2).\\
&\quad (z_1 = \mathbf{0}) = (z_2 = \mathbf{0}) \land\\
&\quad x_1 \otimes z_2 = x_2 \otimes z_1 \land y_1 \otimes z_2 = y_2 \otimes z_1)
\end{aligned}
$$

lemma (**in** *field*) *make-affine-proj-eq-iff*:

$$
\begin{aligned}
&\textit{in-carrierp}\ p \Longrightarrow \textit{in-carrierp}\ p' \Longrightarrow\\
&\ \textit{proj-eq}\ p\ p' = (\textit{make-affine}\ p = \textit{make-affine}\ p')
\end{aligned}
$$

Specification of Point Addition – SPARK Part

```
function Point_Add_Spec
  (M, A, X1, Y1, Z1, X2, Y2, Z2, X3, Y3, Z3 : Math_Int.Math_Int)
  return Boolean
  with Ghost, Import, Global => null;

procedure Point_Add
  (X1, Y1, Z1 : in     Bignum.Big_Int;
   X2, Y2, Z2 : in     Bignum.Big_Int;
   X3, Y3, Z3 :    out Bignum.Big_Int;
   ...
   A          : in     Bignum.Big_Int;
   M          : in     Bignum.Big_Int;
   M_Inv      : in     Types.Word32)
  with
    Depends => ...
    Pre => ...
    Post =>
      Point_Add_Spec
        (Bignum.Num_Of_Big_Int (M, M_First, X1_Last - X1_First + 1),
         ...);
```

Specification of Point Addition – Isabelle Part

definition *point-add-spec* :: *math-int* ⇒ *math-int* ⇒
math-int ⇒ *math-int* ⇒ *math-int* ⇒ *math-int* ⇒ *math-int* ⇒
math-int ⇒ *math-int* ⇒ *math-int* ⇒ *math-int* ⇒ *bool*
where
point-add-spec m a x_1 y_1 z_1 x_2 y_2 z_2 x_3 y_3 z_3 =
(*let r* = *residue-ring* (*int-of-math-int m*);
a' = *int-of-math-int a mod int-of-math-int m*
in cring.proj-eq r
(*cring.padd r a'*
(*int-of-math-int* x_1, *int-of-math-int* y_1, *int-of-math-int* z_1)
(*int-of-math-int* x_2, *int-of-math-int* y_2, *int-of-math-int* z_2))
(*int-of-math-int* x_3, *int-of-math-int* y_3, *int-of-math-int* z_3))

why3-consts
Lsc--ec--point-add-spec.point-add-spec = *point-add-spec*

Agenda

1. Introduction
2. A Link between Why3 and Isabelle
3. Applications
4. **Conclusion**

Achievements

- Correctness of complex mathematical algorithms can be proved using SPARK

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- Behaviour of programs can be specified in an abstract way
- SPARK implementation can be shown to correspond to abstract specification

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  e.g. goal matching algorithm sometimes gets confused by
  complex control flow

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- Continued verification of security-critical components and protocols

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