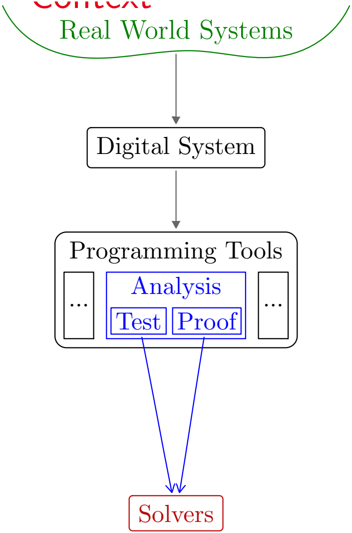
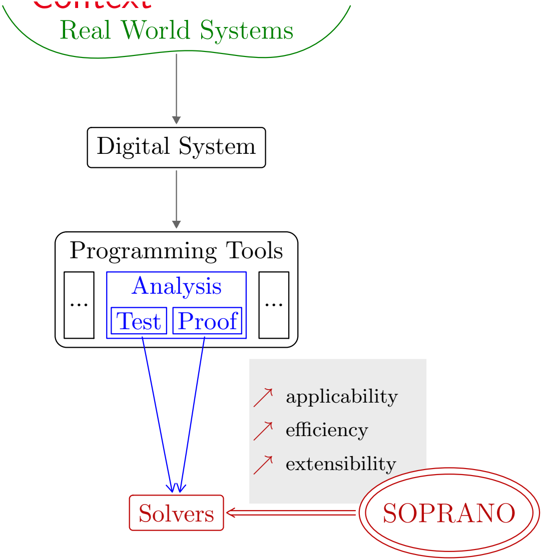
Context



Real World Systems
Digital System
Programming Tools
...
Analysis
Test
Proof
...
Solvers

Context


Real World Systems
Digital System
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...
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applicability
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SOPRANO

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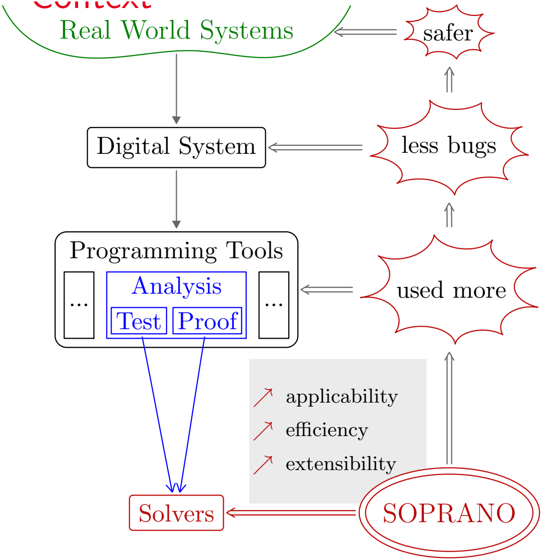
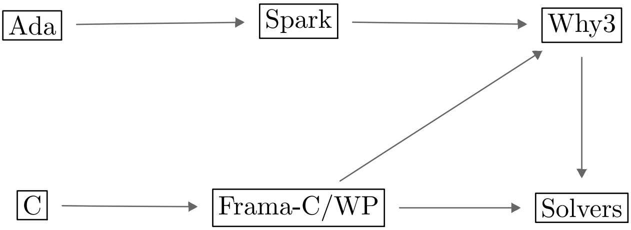

Real World Systems
safer
Digital System
less bugs
Programming Tools
...
Analysis
Test
Proof
...
used more
applicability
efficiency
extensibility
Solvers
SOPRANO

Toolchains



Challenges

Currently uses SMT solvers:

- ✔ Good handling of arithmetic (integers, bitvectors, reals)
- ✔ Good handling of axioms
- ✔ Reasonably fast
- ✔ Agreed semantics between all provers (SMTLIB)

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- ✘ Many properties involving x/y, $x \times y$, x^y, $x \bmod y$, $x \operatorname{rem} y$
- ✘ Most properties involving floating-point values
- ✘ Properties involving conversions between types (integers $\leftrightarrow$ bitvectors, integers $\leftrightarrow$ reals, integers $\leftrightarrow$ floats)

Floating Points

- ✔ Clear Semantic: $x \oplus y = \circ(x + y)$
- ✘ Few algebraic properties: not associative, $x \oplus y = x \not\Rightarrow y = 0$
- ✘ Counter-intuitive: $\overbrace{0.1 \oplus \cdots \oplus 0.1}^{10} \neq 0.1 \otimes 10. = 1.$
- ✘ State of the art: current bit-blasting doesn't scale
- ✘ Pervasives in programs

Interesting and Simple Real Examples

```
/*@ requires 0 ≤ x ≤ 1000;
    requires 0 ≤ y ≤ 1000;
    ensures  0 ≤ \result ≤ 1;   @*/
double x_normalisation(double x,double y){

  return x/sqrt(x*x + y*y);

}
```

Domain Specific Approach of CP

$$X_i \in [1;10] \implies X_0 \oplus X_1 \oplus X_2 \oplus X_3 \oplus X_4 \oplus X_5 \oplus X_6 \oplus X_7 \in [8;80]$$

Z3 (SMT): 3s
COLIBRI (CP): 0.1s

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$$X_i \in [1;10] \implies X_0 \otimes X_1 \otimes X_2 \otimes X_3 \otimes X_4 \otimes X_5 \otimes X_6 \otimes X_7 \in [1;10^8]$$

Z3 (SMT): 31min
COLIBRI (CP): 0.1s

COLIBRI: Floating Point (Bruno Marre)

- Precise domain propagation:
  $x \oplus y = 0.05 \implies x, y \in [-0.1259..; 0.175....]$

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  $o(f(x)) < o(y) \implies o(x) \leq o(f^{-1}(o(y)))$

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- Instantiated for many functions

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- Linearization of constraints for simplex

COLIBRI: Example of Reasoning

$$0 \leq x, y \leq 1000 \implies \sqrt{x^2 \oplus y^2} \geq x \ ?$$

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$$\mathrm{o}(x^2) + \mathrm{o}(y^2) \leq \mathrm{o}(x^2)$$

$$\mathrm{o}(x^2) + \mathrm{o}(y^2) = \mathrm{o}(x^2)$$

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$x < x$ if $\mathrm{o}(x^2)$ is normalized

$\mathrm{o}(x^2)$ is denormalized

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$x < x$ if $\mathrm{o}(x^2)$ is normalized

$\mathrm{o}(x^2)$ is denormalized

x the minimum of the remaining values is a solution

There is a counter-example!

Interesting and Simple Real Examples: Corrected

```
/*@ requires 0.0001 ≤ x ≤ 1000;
    requires 0.0001 ≤ y ≤ 1000;
    ensures  0 ≤ \result ≤ 1;    @*/
double x_normalisation(double x,double y){

  return x/sqrt(x*x + y*y);

}
```

Other Examples: From SPARK User Rule

```
procedure User_Rule_7 (X, Y, Z, A : Float;
                       Res        : out Boolean)
 is
 begin
    pragma Assume (Z ≥ 0.0);
    pragma Assume (X ≥ Y);
    pragma Assume (Y ≥ Z);
    pragma Assume (X > Z);
    pragma Assume (A ≥ 1.0);
    Res := (X − Y) / (X − Z) ≤ A;
    pragma Assert (Res);      −− valid
 end User_Rule_7;
```

Other Examples: From SPARK User Rule

$$A \leq \frac{X \ominus Y}{X \ominus Z} \leq B \quad \text{with} \dots$$

$$\sqrt{X^2 \ominus Y^2} \leq X \quad \text{with} \dots$$

$$\frac{X}{\sqrt{X^2 \oplus Y^2}} \leq 1 \quad \text{with} \dots$$

Floating-Point Arithmetic: Alt-Ergo
(OCamlPro, UPSud)

Floating-point rounding operator on rational constants; Interval domains

```
axiom rounding_operator_1 :
   forall  x :  real .
   forall  i , j  :  real .
   forall  md : fpa_rounding_mode.
   forall  p,m :  int
    [round(m,p,md,x), x in  [i , j ]].
      i  ≤ x ≤ j →
      round(m,p,md,i) ≤ round(m,p,md,x) ≤ round(m,p,md,j)
```

(Kailiang Ji, post-doc SOPRANO)

Floating-Point Arithmetic: Recap

Progression of COLIBRI and Alt-Ergo on AdaCore benchmarks:

	Before SOPRANO	Current Results
COLIBRI	18 / 28	25 / 28
Alt-Ergo	2 / 28	19 / 28

Bitvector and Integer Arithmetic (Chihani)

COLIBRI:

- High-level view of bitvectors
- New propagations for integers $\leftrightarrow$ bitvectors

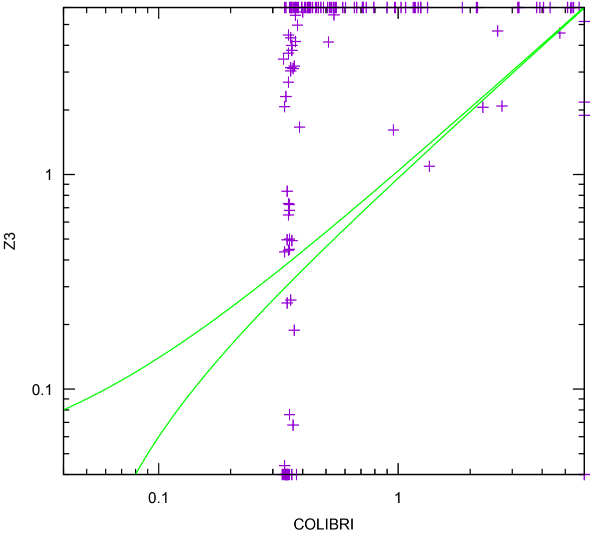
Evaluations & Diffusion

- AdaCore examples

Evaluations & Diffusion

- AdaCore examples
- Participation at SMT-COMP 2017

COLIBRI on SMT-COMP FP category



Evaluations & Diffusion

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Evaluations & Diffusion

- AdaCore examples
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COLIBRI: Freeware For Research

F. Bobot, Z. Chihani, M. Iguernlala, and B. Marre.
Fpa solver.
Technical report, ANR SOPRANO, ANR-14-CE28-0020, 2016.
http://soprano-project.fr/downloads/D3_1.pdf.

Z. Chihani, B. Marre, F. Bobot, and S. Bardin.
Sharpening constraint programming approaches for bit-vector theory.
In *CPAIOR*, 2017.

S. Conchon, M. Iguernlala, K. Ji, G. Melquiond, and C. Fumex.
A three-tier strategy for reasoning about floating-point numbers in smt.
In *Computer Aided Verification*, 2017.

Floating-Point Arithmetic: Monotonic functions (CEA, UPSud)

Theorem

Let $D, E \subset \mathcal{R}$, $f : D \mapsto E$ and $f^{-1} : E \mapsto D$ such that

- $\forall x : D,\ f^{-1}(f(x)) = x$
- *f increasing*

We have

- $\forall x \in D, \mathrm{o}(y) \in E, \mathrm{o}(f(x)) < \mathrm{o}(y) \implies \mathrm{o}(x) \leq \mathrm{o}(f^{-1}(\mathrm{o}(y)))$
- $\forall x \in D, y \in E, \mathrm{o}(f(x)) < \mathrm{o}(f(y)) \implies x < y$

Instantiated for many functions in COLIBRI's DBM

Interesting and Simple Real Examples

```
/*@ ensures \result ≤ (double) 1; @*/
double test2(){
  double x = read_sensor();
  /*@ assert (double) 0 ≤ x ≤ (double) 1000; @*/
  double y = read_sensor();
  double z = read_sensor();

  x = x * x + z * z + y * y + 1;

  if (z ≤ y){
    return (x−y)/(x−z);
  } else {
    return (x−z)/(x−y);
  }
}
```

The Problems